Physical controls on sediment deposition, erosion, and transport

David J. Topping and Paul E. Grams (USGS-GCMRC)

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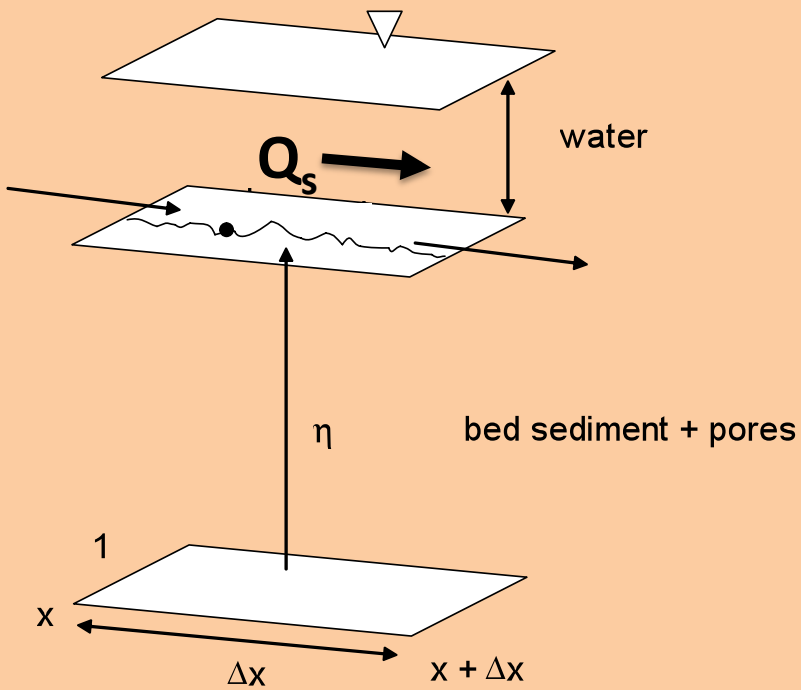
Equation that relates changes in bed elevation to the sediment-transport field in the flow is the sediment mass-conservation relation of Exner (1920, 1925)



$$\frac{\partial \eta}{\partial t} = -\frac{1}{0.65}\left(\frac{\partial V_S}{\partial t} + \nabla \bullet \mathbf{Q_s}\right)$$

Colby (1964)
Rubin and Hunter (1982)
Topping and others (2000b)

What is $\nabla \cdot \mathbf{Q}_S$?

- $\nabla \cdot \mathbf{Q}_S$ is the spatial change in the sand flux

- Sand flux is the integral product of the suspended-sand concentration and velocity

So $\nabla \cdot \mathbf{Q}_S$ is the spatial change in the amount of sand transported in the water column

On the basis of Colby (1964)...

Given a sufficient upstream supply of sand, because of differences arising from how the flow interacts with different bed topographies, some locations on the bed scour (erode) as discharge increases and some locations fill (deposit).

In addition, because of differences arising from how the flow interacts with the same bed topography at different discharges, the same location may scour at some discharges and fill during other discharges.

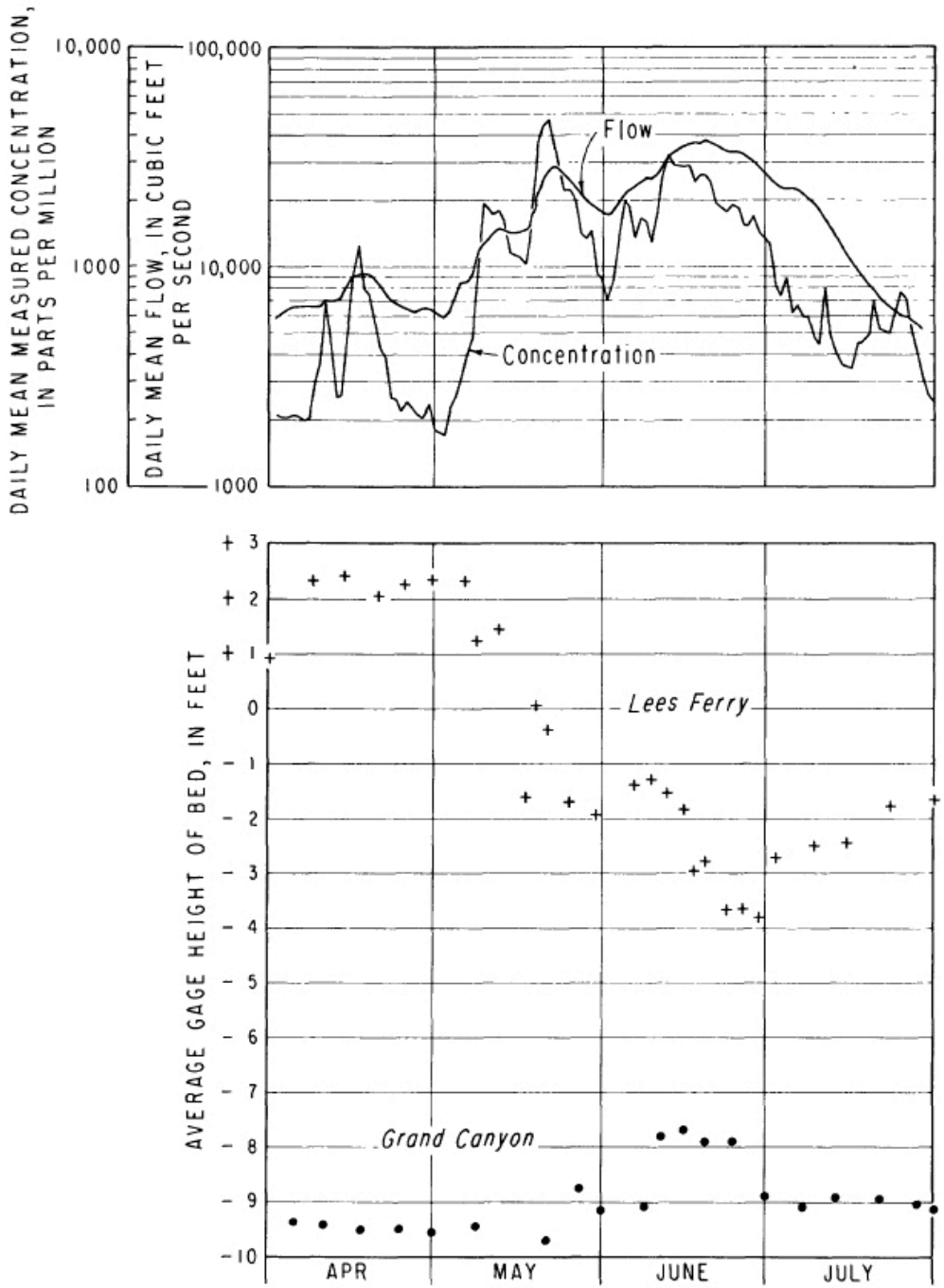
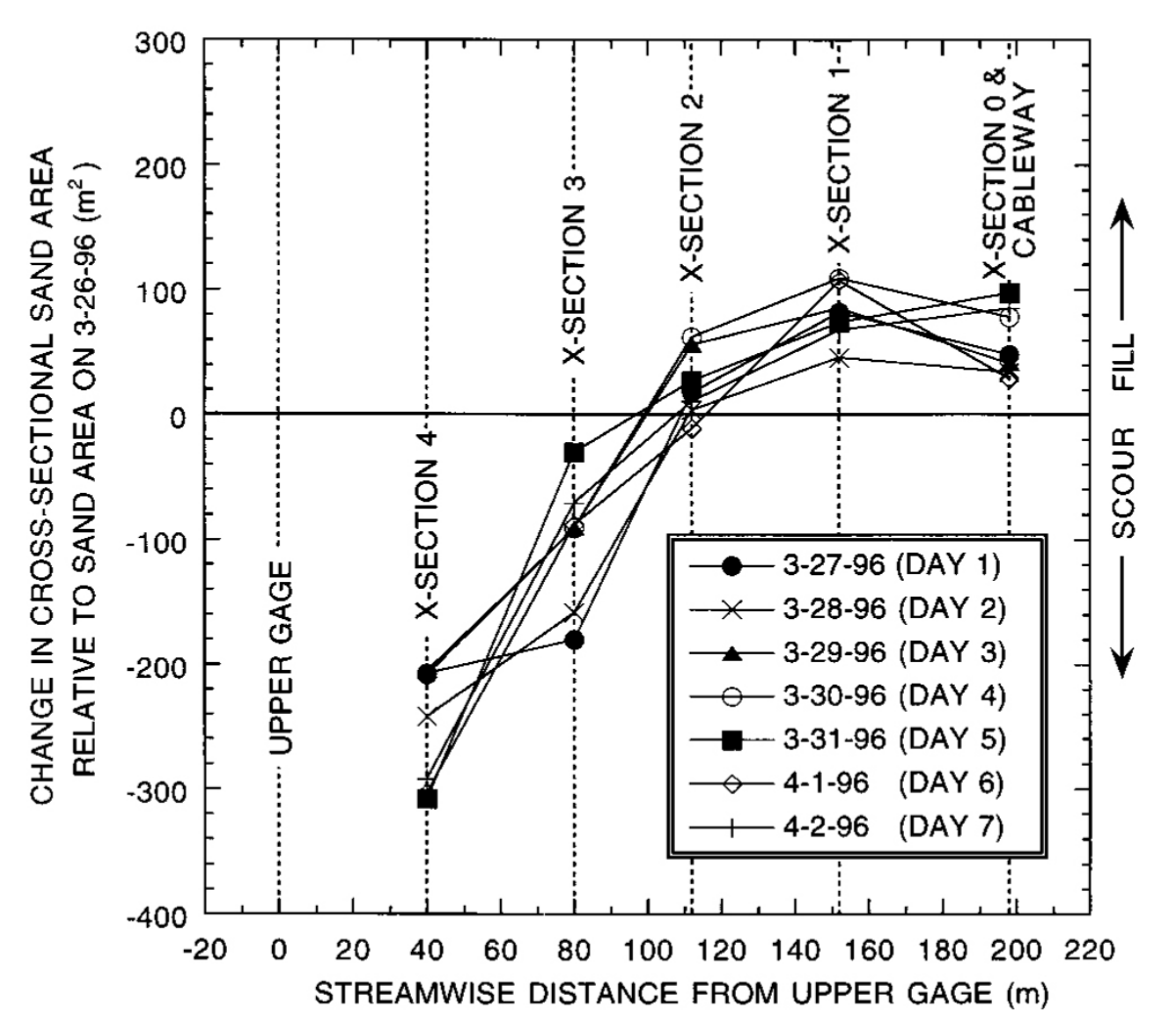
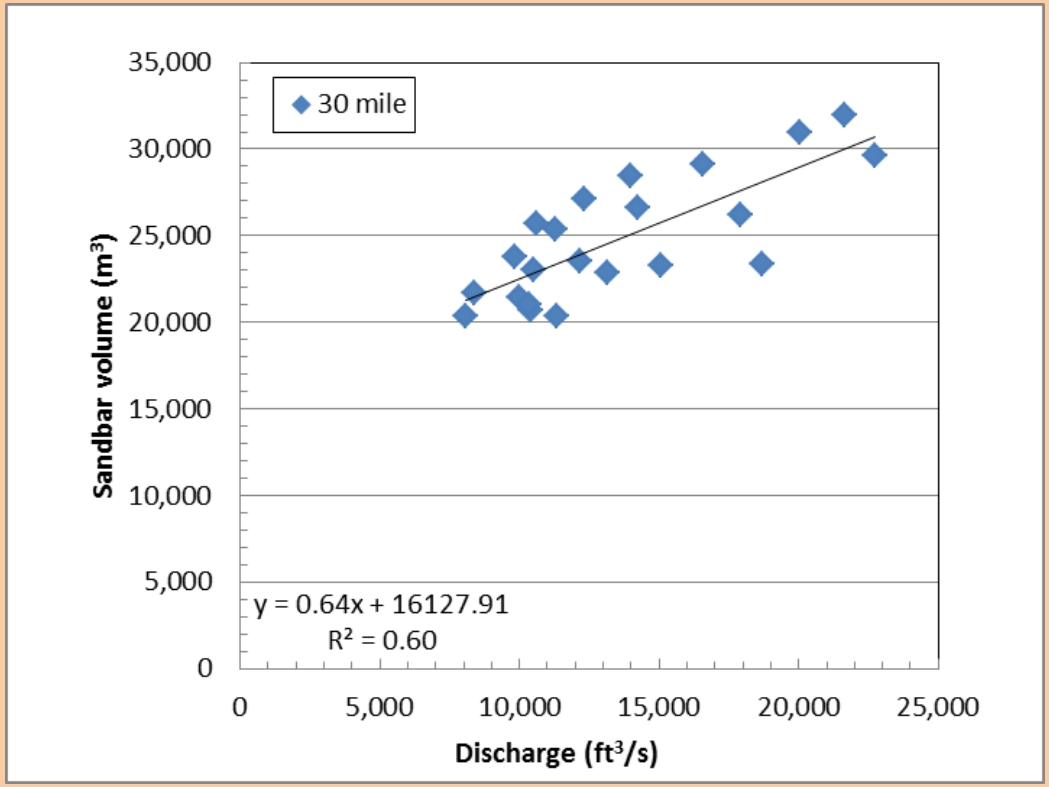


FIGURE 18.—Changes of average bed position at Lees Ferry and near Grand Canyon, Ariz., April–July 1959. Concentration and flow near Grand Canyon.

From Colby (1964).



As discharge increased from 8,000 to 45,000 cfs during the 1996 controlled flood, upstream part of reach scoured while lower part of reach filled because $\nabla \cdot \mathbf{Q}_S$ is different at different discharges (after Topping and others, 2000a, 2000b).



Discharge control of the amount of sand that may deposit at a given location (through changes in $\nabla \cdot \mathbf{Q_S}$ with changing discharge) make sandbar-based inferences of the long-term effects of dam operations on the amount of sand in the Colorado River in Grand Canyon problematic. So long as there is an upstream supply of sand, some sandbars tend to gain sand with increasing discharge while others (not shown) tend to lose sand with increasing discharge. In this example, 60% of the variance is explained by differences in the discharge of water antecedent to the sandbar survey (Paul Grams, USGS, later today).

But $\nabla \bullet \mathbf{Q_s}$ does not depend only on flow conditions

$$\mathbf{Q_s} \approx \langle C_s \mathbf{U} \rangle h$$

sand concentration velocity flow depth

- $\nabla \bullet \mathbf{Q_s}$ depends on spatial changes in sand concentration, velocity, and flow depth

- Sand concentration depends on both flow conditions and bed-sand conditions

On the basis of Rouse (1933), Engelund and Hansen (1967) Hunt (1969), Smith (1977), McLean (1992), Rubin and Topping (2001, 2008), and Topping and others (2010)...

$$C_S \propto u_*^{3.5} A_b D_b^{-2.5}$$

FLOW CONDITIONS

shear velocity
"flow strength"

BED-SAND CONDITIONS*

sand area

sand grain-size
distribution

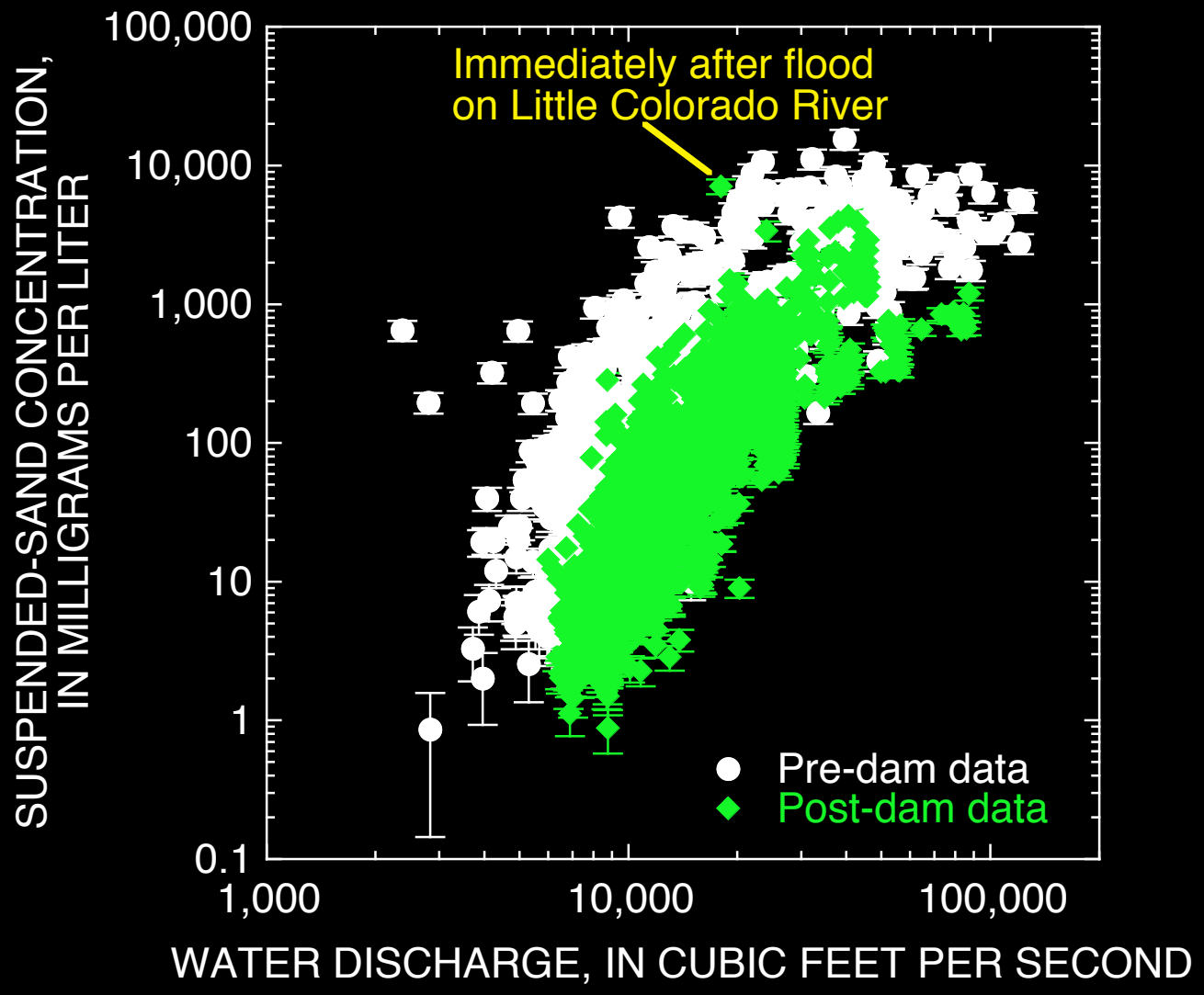
*Bed-sand conditions are determined by interaction of flow, bed topography, upstream sand supply, and pre-existing bed-sand conditions...

...solved by application of sediment advection-diffusion equation with lower flux boundary condition...very complicated

$$C_S \propto u_*^{3.5} A_b D_b^{-2.5}$$

- Flow conditions raised to a power of 3.5
  - factor of 2 INCREASE in flow conditions leads to factor of 11 INCREASE in sand concentration
- Bed-sand area (amount of sand) raised to a power of 1
  - factor of 2 INCREASE in amount of sand covering the bed leads to factor of 2 INCREASE in sand concentration
- Bed-sand grain-size distribution (coarse vs. fine) raised to a power of -2.5
  - factor of 2 INCREASE in bed-sand median grain size leads to factor of 6 DECREASE in sand concentration
  - we have observed up to a factor of 9 change in bed-sand median grain size over time in the Colorado River....this change leads to > factor of 200 change in sand concentration
  - this coupled with only a factor of 5 change in the amount of sand on the bed leads to the observed factor of 1,000 variation in sand concentration

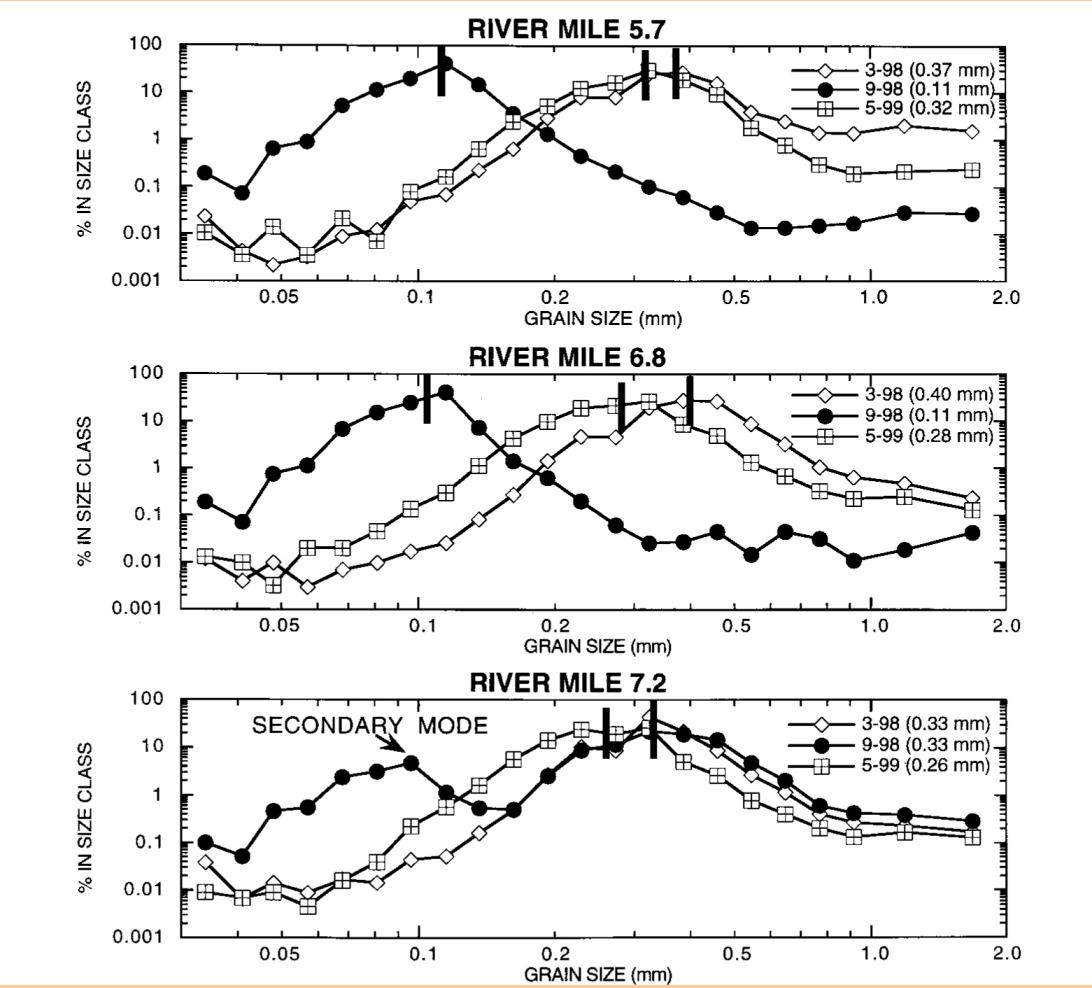
1944-2010 EDI and EWI measurements at Grand Canyon gage



(after Rubin and others, 2002)

300-400x increase in silt and clay concentration, 10-20x increase in sand concentration over 2 hours 30 km downstream from flooding tributary

Only 2-3% increase in the discharge of water!!!

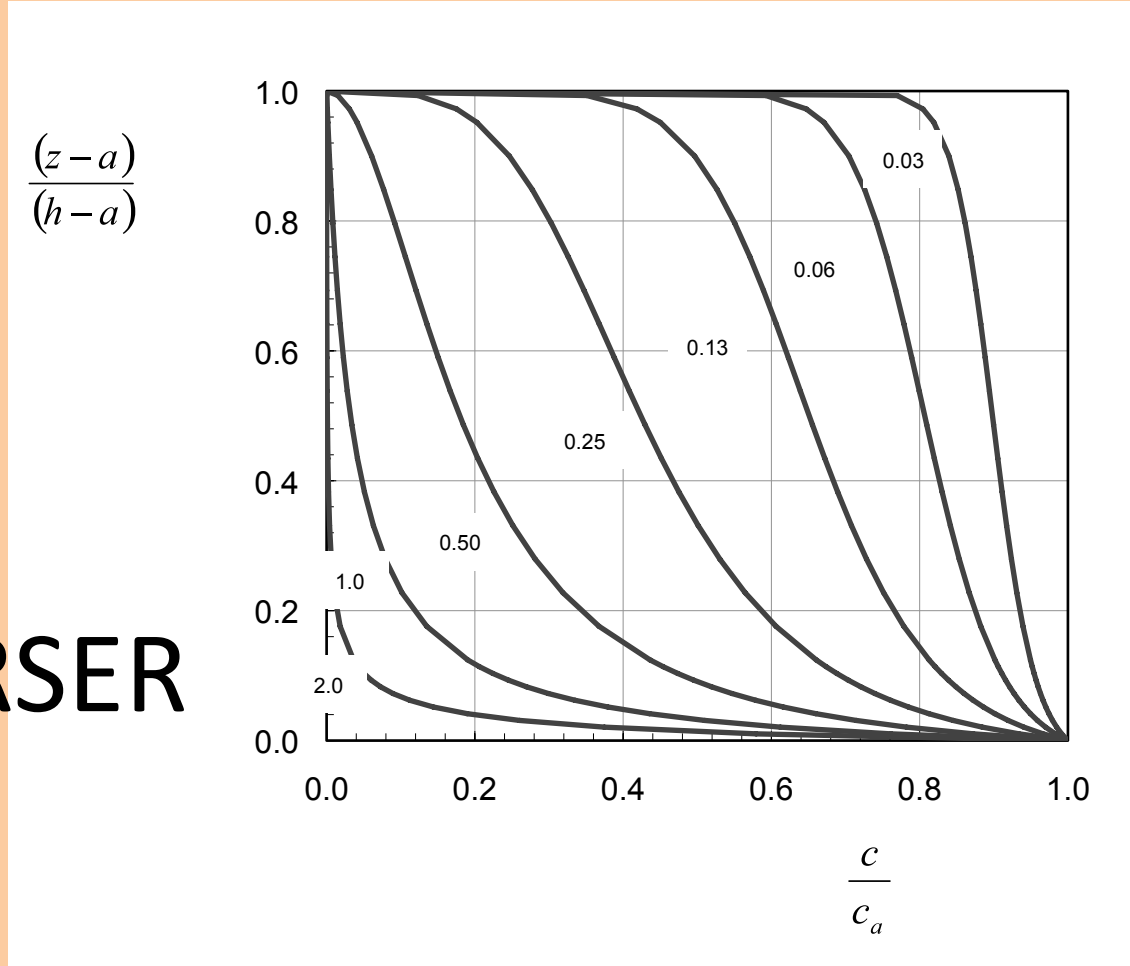


Examples of changes in bed-sand grain-size distribution in upper Marble Canyon associated with large Paria floods (FINING) and subsequent dam operations (COARSENING). Changes in bed-sand grain size have been detected over 40-50 miles downstream from flooding tributary well within 1 week.

Topping and others (2000b)

Why does bed-sand grain size change so fast?

FINER



COARSER

Finer sand is carried higher in the flow where velocities are highest (Rouse, 1933)....also why finer sand will be depleted first when supply is limited.

Back to Exner (1920, 1925)

$$\frac{\partial \eta}{\partial t} = -\frac{1}{0.65}\nabla \bullet \mathbf{Q_s}$$

Because

$$\mathbf{Q_s} \approx \langle C_S \mathbf{U} \rangle h$$

$$C_S \propto u_*^{3.5} A_b D_b^{-2.5} \qquad u \propto u_*$$

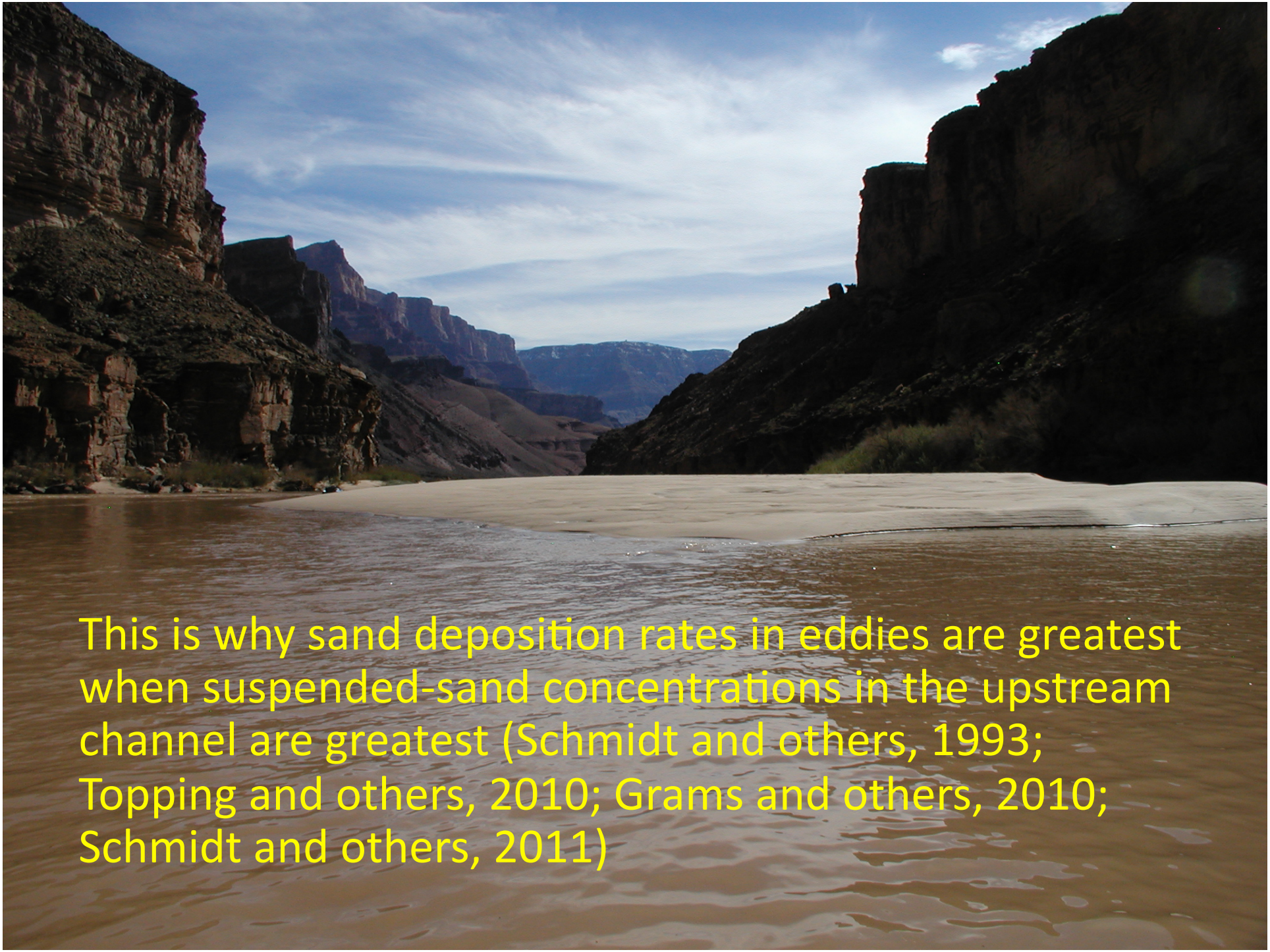
Therefore

$$\frac{\Delta \eta}{\Delta t} \propto -\frac{\Delta(\langle C_S u \rangle h)}{\Delta x} \propto -\frac{\Delta\left(u_*^{4.5} h A_b D_b^{-2.5}\right)}{\Delta x}$$

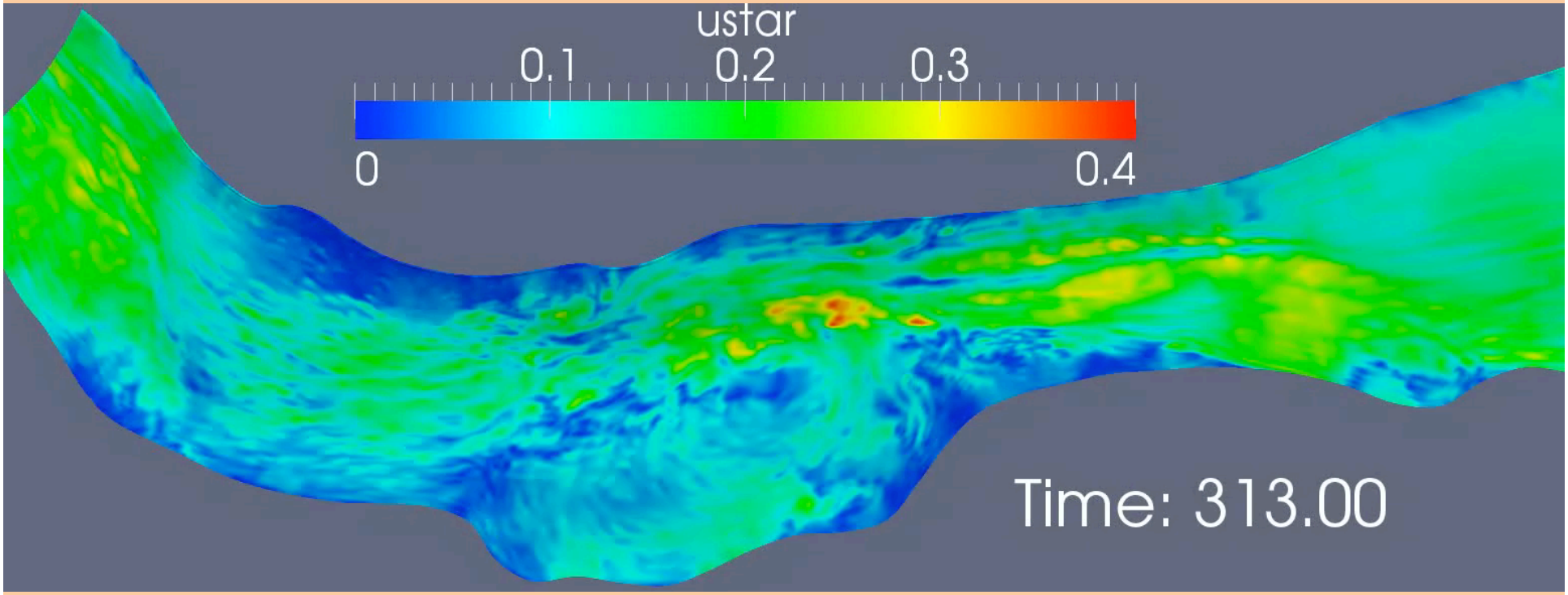
FLOW BED SAND

$$\frac{\Delta\eta}{\Delta t} \propto -\frac{\Delta\left(u_*^{4.5} h A_b D_b^{-2.5}\right)}{\Delta x}$$

- Spatial decrease in “flow” leads to deposition
- Spatial increase in bed-sand grain size leads to deposition
- Spatial decrease in bed-sand area (amount) leads to deposition
- Greatest deposition rates occur in eddies when greatest flow “deceleration” occurs between channel and eddy, and sand in upstream channel is as fine as possible and amount on upstream channel bed is relatively large


This is why sand deposition rates in eddies are greatest when suspended-sand concentrations in the upstream channel are greatest (Schmidt and others, 1993; Topping and others, 2010; Grams and others, 2010; Schmidt and others, 2011)

$$\frac{\Delta\eta}{\Delta t} \propto -\frac{\Delta(\langle C_S u\rangle h)}{\Delta x} \propto -\frac{\Delta\left(u_*^{4.5} h A_b D_b^{-2.5}\right)}{\Delta x}$$

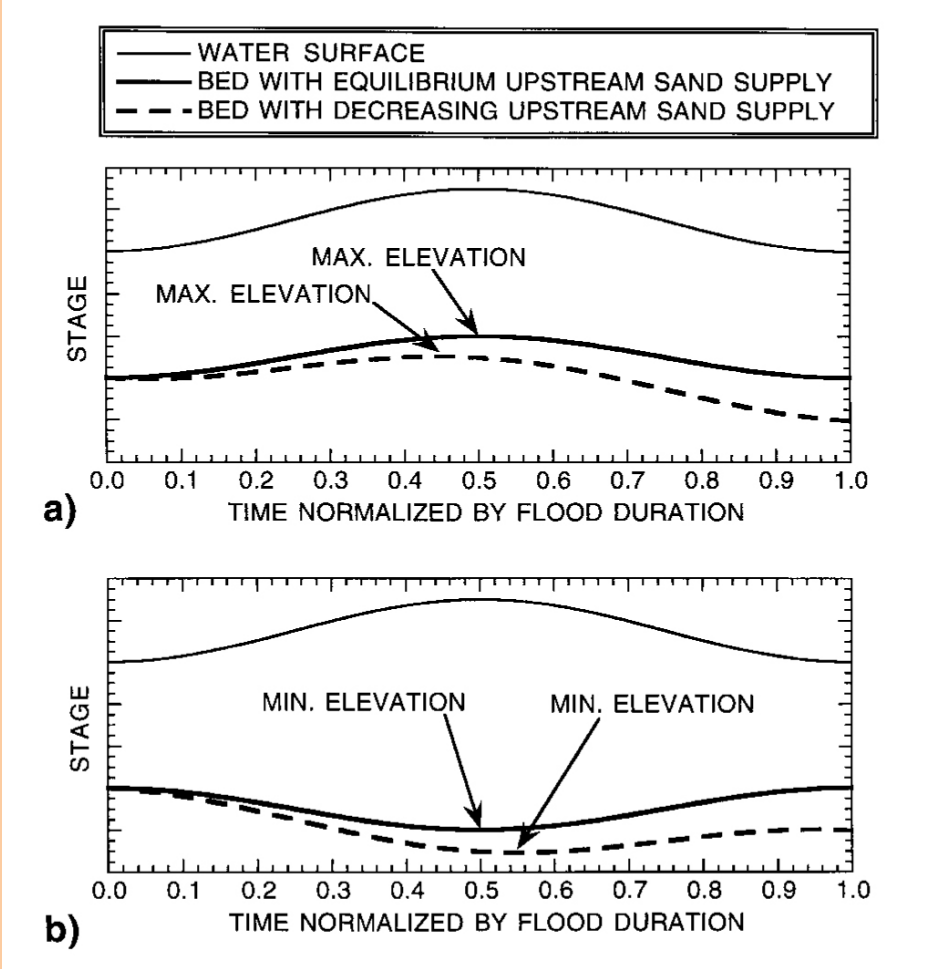


LES model run from Mark Schmeeckle (ASU)

Caution

If upstream sand supply gets too depleted (i.e., SMALL SAND AREA AND COARSE SAND GRAIN-SIZE DISTRIBUTION ON CHANNEL BED), environments where sand normally deposits by virtue of flow "deceleration", i.e. eddies, can erode!

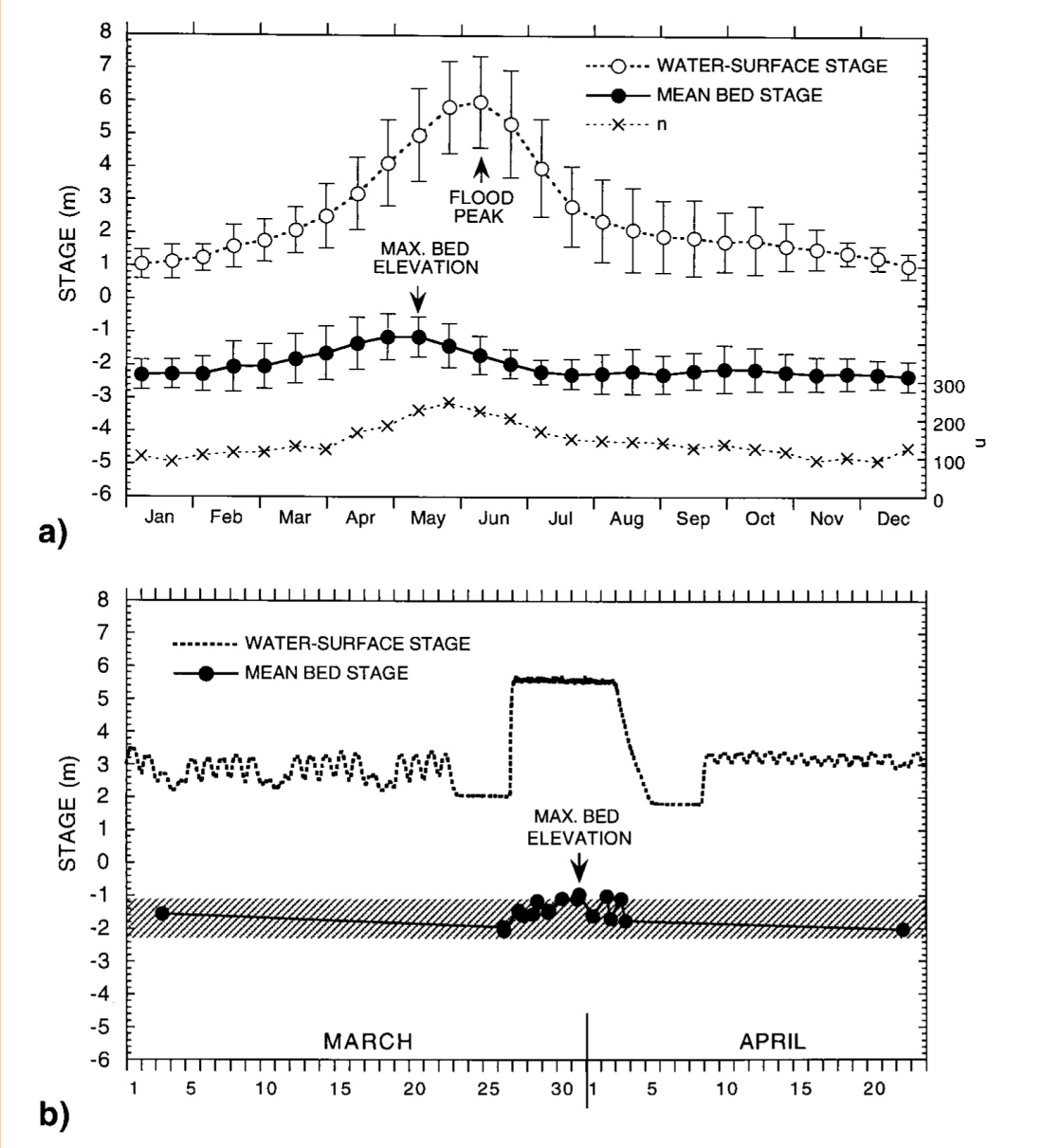
(Paul Grams will show examples later today.)



Negative $\nabla \cdot \mathbf{Q_s}$ with increasing discharge

Positive $\nabla \cdot \mathbf{Q_s}$ with increasing discharge

A decrease in the upstream sand supply can offset the tendency for sand to deposit in certain environments during floods...in other words... places that normally gain sand during a flood may gain little or even lose sand if the upstream sand supply becomes too depleted (Topping and others, 2000b).



Progressive depletion of the upstream sand supply during pre-dam snowmelt floods and the 1996 controlled flood caused the aggrading bed at the Grand Canyon gage cableway to scour (erode) prior to the recessions of these floods (Topping and others, 2000b).

In the previous slides there is no distinction made between new tributary supplied sand and background sand storage, both are combined in D_b and A_b.

Thus, if background sand storage in a reach of the Colorado River decreases in volume or coarsens over time, then similar magnitudes of tributary sand inputs will result in progressively lower eddy-sandbar deposition rates over a series of artificial floods released from Glen Canyon Dam.

Conversely, if background sand storage remains constant over time, then similar magnitudes of tributary sand inputs will result in similar eddy-sandbar deposition rates over a series of artificial floods released from Glen Canyon Dam.

And, if background sand storage in a reach of the Colorado River increases in volume or fines over time, then similar magnitudes of tributary sand inputs will result in progressively higher eddy-sandbar deposition rates over a series of artificial floods released from Glen Canyon Dam.

Implications for monitoring

- Because both new tributary supplied sand and sand in background storage combine to provide the upstream sand supply to rebuild sandbars during artificial floods, it is critical to have a method for monitoring long-term (decadal) effects of dam operations on the sand budget (described in the TWG-approved core monitoring plan for Goal 8).

- Because of the accumulating potential biases in the mass-balance approach (reviewed tomorrow), this method must be a geomorphic-based approach.

The paradox is that even though sandbars must be monitored to evaluate their condition, this long-term sand budgeting method cannot be based on only monitoring sandbars because the effects of changing $\nabla \cdot \mathbf{Q_s}$ with changing discharge may exert too strong a control on the volume of sand that may be stored in many sandbars at any given discharge, thus potentially masking changes in the long-term sand budget.

"Some eddies are really good sand traps at high discharge while others are really good sand traps at low discharge."

(Paul Grams, later today)

Ancillary slides

Spatial interactions of flow with bed topography are dominant controller of magnitudes and locations of deposition and erosion

By virtue of analyses in Colby (1964), Rubin and Hunter (1982), and Topping and others (2000b), changes in bed elevation attributable to $\partial V_S / \partial t$ are much smaller than those attributable to $\nabla \bullet \mathbf{Q_S}$. For example, during the 1996 controlled flood at the Grand Canyon gaging station, only 1 cm of bed scour would be needed to equal the amount of sand in suspension by $\partial V_S / \partial t$, whereas, as a result of $\nabla \bullet \mathbf{Q_S}$, the bed aggraded by 0.5 m under the cableway.

Let's take the simplified version of the Exner equation,

$$\frac{\partial \eta}{\partial t} = -\frac{1}{0.65} \nabla \bullet \mathbf{Q_s}$$

and get rid of the cross-stream dimension to make the physics as simple as possible...

$$\frac{\partial \eta}{\partial t} = -\frac{1}{0.65} \left(\frac{\partial q_S}{\partial x} \right)$$

The unit sand flux, q_S, is the integral of the product of suspended-sand concentration, C_S, and downstream velocity, u

$$q_s = \int_o^h (C_S u) dz$$

We will explore the controls on C_S next.

In reality (time-varying non-uniform flow with changing upstream sand supply), C_S is in two dimensions (downstream and vertical) really complicated...

$$\frac{\partial C_S}{\partial t} + \frac{\partial \left(uC_S\right)}{\partial x} - \frac{\partial \left(w_S C_S \left(1 - C_S\right)\right)}{\partial z} = \frac{\partial}{\partial x}\left(2K\frac{\partial C_S}{\partial x}\right) + \frac{\partial}{\partial z}\left(2K\frac{\partial C_S}{\partial z}\right)$$

We'll simplify...

For an equilibrium upstream supply of sand, and steady, uniform flow (with a logarithmic velocity profile), for *m* size classes of sand *S*:

$$\left(\frac{C_m}{1-C_S}\right)_z = \left(\frac{C_m}{1-C_S}\right)_a \left[\left(\frac{a}{z}\right)\left(\frac{h-z}{h-a}\right)\right]^p$$

After Rouse (1933), Hunt (1969), and Smith (1977).

Lower boundary condition is a flux boundary condition regulated by a reference concentration relation where $(C_m)_a$ depends on boundary shear stress (flow) and bed-sand grain-size distribution and area (upstream sand supply).

Exponent *p* (Rouse number) depends on flow and settling velocity of sand size-class *m*.

On the basis of Engelund and Hansen (1967) and McLean (1992), Rubin and Topping (2001) and Topping and others (2010) arrived at the following proportionalities...

First

$$C_S \propto u_*^{3.5} A_b D_b^{-2.5}$$

D_b is the median grain size of the bed sand

A_b is the fractional amount of sand covering the bed

u_* is the shear velocity (a measure of flow strength) and is defined as $\sqrt{\frac{\tau_b}{\rho}}$

Because reach-averaged u_* is directly proportional to discharge, it is obvious from

$$C_S \propto u_*^{3.5} A_b D_b^{-2.5}$$

why, for constant D_b and A_b (constant upstream sand supply) and the same daily volume of water released from the dam, fluctuating flows transport more sand than steady flows.

Because $q_s = \int_o^h (C_S u) dz$ can be approximated as

$$q_S \approx \langle C_S u \rangle h$$

$\frac{\partial \eta}{\partial t} = -\frac{1}{0.65}\left(\frac{\partial q_S}{\partial x}\right)$ can be approximated as

$$\frac{\Delta \eta}{\Delta t} \propto -\frac{\Delta(\langle C_S u \rangle h)}{\Delta x}$$

Because $C_S \propto u_*^{3.5} A_b D_b^{-2.5}$ and $u \propto u_*$

$$\frac{\Delta\eta}{\Delta t} \propto -\frac{\Delta(\langle C_S u \rangle h)}{\Delta x} \propto -\frac{\Delta\left(u_*^{4.5} h A_b D_b^{-2.5}\right)}{\Delta x}$$

Therefore, as concluded by Topping and others (2010), eddy-sandbar deposition rates are greatest not necessarily when the most sand is present on the upstream bed of the channel (that is, only channel A_b is maximized) but when a large quantity of fine sand is present on the upstream bed of the channel (that is, D_b is minimized and A_b is reasonably large).

Finally, let's examine $$\frac{\Delta\eta}{\Delta t} \propto -\frac{\Delta q_S}{\Delta x} - \frac{\Delta\left(u_*^{4.5} h A_b D_b^{-2.5}\right)}{\Delta x}$$

to investigate the effects of a decreasing upstream sand supply on erosion and deposition in locations where the flow interacts with bed topography to produce a strong convergence (deposition) or divergence (erosion) in sand flux during floods.

Because even though the strongest control on the deposition (or erosion) rate $\frac{\Delta\eta}{\Delta t}$ at a given location is the spatial change in flow conditions (through the $u_*^{4.5} h$ term)

substantial convergence or divergence in the sand flux can be driven by a spatial change in sand supply (through the $A_b D_b^{-2.5}$ term).