REVISION OF THE PROCEDURE TO COMPUTE SEDIMENT DISTRIBUTION IN LARGE RESERVOIRS

Sedimentation Section
Hydrology Branch

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REVISION OF THE PROCEDURE TO COMPUTE SEDIMENT DISTRIBUTION IN LARGE RESERVOIRS

INTRODUCTION

The procedure outlined herein is a revision to one presented earlier by Borland and Miller (1960) on the computation of sediment distribution in large reservoirs. Moody (1962) developed this revised procedure which entails (1) the elimination of the trial and error method to determine the depth of sediment deposited at the dam, and (2) the revision of the existing design curves of the four different types of reservoirs. Included in this report are the description of the procedure developed by Moody and a detailed example of the computational process.

TECHNICAL DEVELOPMENT OF THE REVISED PROCEDURE

The current procedure by Borland and Miller (1960) to compute the sediment distribution in reservoirs was revised by developing a graphical method based on mathematics as will be subsequently described. A more direct procedure to determine the depth of sediment deposited at the dam resulted from Moody's work. This eliminated the present cut and try method which may involve sometimes as many as five trials. The existing design curves were also improved and revised by using the method of least squares to obtain a better fit to the observed data.

The basic equation to develop the revised procedure is expressed as follows:

$$S = \int_{o}^{y_o} A\,dy + \int_{y_o}^{H} Ka\,dy \qquad (1)$$

where

- S = total sediment in the reservoir
- o = the original zero elevation of the dam
- y_o = the zero elevation of the dam after sediment inflow, usually taken as the 100-year period
- A = reservoir surface area
- dy = incremental depth
- H = total depth of reservoir commonly determined by the normal water surface

K = a constant of proportionality for converting relative areas to actual areas for a given reservoir

a = relative area

Moody's further treatment of the above equation by integration and simplification through a series of intermediate steps resulted in the expression below:

$$\frac{1 - v_o}{a_o} = \frac{S - V_o}{H\ A_o} \qquad (2)$$

where

v_o = relative reservoir volume at zero depth

a_o = relative reservoir area at zero depth

V_o = total reservoir volume at zero depth

H = depth of reservoir

A_o = total reservoir area at zero depth

The above equation permits the condition for determining y_o, the basic objective of this analysis. To accomplish this most easily, the following definition is made:

$$h(p) = \frac{1 - v(p)}{a(p)} \qquad (3)$$

and

$$h'(p) = \frac{S - V(y)}{HA(y)} \qquad (4)$$

or to emphasize the dependence of V and A on p, and since y = pH

$$h'(p) = \frac{S - V(pH)}{HA(pH)} \qquad (5)$$

where

h = a function of one of four types of theoretical design curves

h' = a function of a particular reservoir and its anticipated sediment storage

Figure 1 shows a plotting of the relationship in Equation (3) above. A curve is shown for each of the four types of reservoirs. The fifth curve plotted represents this relationship for another procedure of computing the sediment distribution known as the Area-Increment Method fully described in Borland and Miller's paper (1960).

By Equation (2) the expressions of Equations (3) and (5) must be equal to determine the depth of sediment at the dam. This can be done graphically by plotting the relationship expressed by Equation (5) and superposing it upon the proper type curve of Equation (3), the intersection defines p_o, from which y_o can be calculated. The term p_o is the relative depth measured from the bottom of the reservoir at height, y_o. An example of the foregoing procedure will be presented later in this report.

The revised analytical storage and area design curves given in Figures 2, 3, 4, 5, and 6 are defined by

$$v(p) = \frac{1}{\beta(m+1,\ n+1)} \int_0^p p^m (1 - p)^n \, dp \tag{6}$$

and

$$a(p) = \frac{p^m (1-p)^n}{\beta(m+1,\ n+1)} \tag{7}$$

The characteristic parameters m and n were determined by fitting curves of the type represented by Equation (6) to the distributions of the curves in Figures 2, 3, 4, and 5. This was accomplished by minimizing the sum of the squares of the deviations of analytical values. The "best" values of m and n were calculated to two decimal places for each type curve.

Moody concluded from his investigation that the curves in Figures 2 to 6, inclusive, supersede those in the Borland and Miller paper. His work also shows that the family of curves in Figure 1 can be used to determine the depth of sediment at the dam as will be illustrated subsequently in this report.

This report was prepared by Joe M. Lara, Hydraulic Engineer, Sedimentation Section. Acknowledgment is made to the work of Mr. Yun-Hsu Lu, Formosan engineer-trainee, who initiated the procedure. Mr. Lu's work was pursued further by A. L. Christensen, formerly of the Sedimentation Section.

EXAMPLE OF A DETAILED COMPUTATION

The computational procedure developed as a result of Mr. Moody's revision will now be presented by steps using the data of the Alamogordo Reservoir, Carlsbad Project, New Mexico. From a plotting of the depth-capacity curve, this reservoir was classified as a Type II using Figure 9 (see Borland and Miller's paper). It is noted in the following procedure that the first five columns of Table 1 are compiled initially. Next, Table 2 is worked out, which combined with Figure 7, furnishes the information necessary to complete the remaining portion of Table 1.

Step 1

In Columns 1, 2, and 3 of Table 1, enter the information pertinent to elevation, original area, and capacity of the reservoir, respectively.

Step 2

The relative depths, expressed decimally, are computed for the corresponding elevations and entered in Column 4. Total depth of the dam, H, is equal to 4,275 - 4,150 = 125 feet; for example, at elevation 4250 the depth is 100 feet, giving a relative depth of 100 ÷ 125 = 0.80.

Step 3

The values of Column 5 are determined by entering the Type II curve in Figure 6 with the values of Column 4. For example, at elevation 4210 feet the relative depth, p, is 0.48 which gives an A_p value of 1.250 from the Type II curve in Figure 6. Note that the figure "II" is entered at the heading of this column, designating the specific classification of Alamogordo Reservoir.

Step 4

The next step in the procedure is to complete the information for Table 2. First, an estimate is made as to establishing a range of elevations where it is judged the "zero" elevation, or depth of sediment at the dam will lie. Data of the lower portion of the reservoir are selected for establishing this range. In this example it was judged that the zero elevation would be within a 40-foot range, therefore the information for elevations from 4150 to 4190 feet was used. Column 1 lists these elevations and Columns 2 and 3 show the corresponding relative depths (same as Column 4, Table 1) and original capacities (same as Column 3, Table 1), respectively.

Step 5

The values in Column 4 are determined by subtracting those of Column 3 from the total sediment inflow, S, equal to 24,600 acre-feet in this example.

Step 6

Column 5 is found by multiplying the height of the dam, H, by the original area corresponding to its specific elevation. For example at elevation 4170 feet, H (125 feet) times $A(pH)$ (100) = 12,500.

Step 7

Column 6 is equal to Column 4 divided by Column 5.

Step 8

The next step is to plot the information of Columns 2 versus 6 superposed on the proper type curve of the graph in Figure 1. In the example this has been done in Figure 7 which is an enlargement of the portion of the Type II curve taken from Figure 1 upon which have been superposed the data of Columns 2 and 6. It is noted that the intersection point of these curves is at $p = 0.25$.

Step 9

The lower portion of Table 2 is filled out as shown. In the example the p_oH value equals 0.25 x 125 = 31.3 which added to the bottom elevation of 4150 gives the elevation of sediment deposited at the dam equal to 4181.3 feet. This elevation is transferred to the bottom of Table 1 under the "Supplement" to provide additional information described in the next step.

Step 10

From the preceding information of Table 2, the remaining portion of Table 1 can now be completed. The same procedure as prescribed by Borland and Miller is used. In this example the relative depth at the zero elevation (4181.3 feet) is 4181.3 - 4150 divided by 125 = 0.25, which is entered at the bottom of the table under

"Supplement," as well as the A_p value of 1.00 read from Figure 6 for this relative depth. A reservoir surface area, A_s, of 212 acres is read off the curve in Figure 8 for an elevation of 4181.3 feet and also entered under "Supplement." Next the K value in the "Supplement" is determined from the relationship A_s/A_p which is equal to 212 ÷ 1.00 = 212 in this example. The same values in Column 2 are used in Column 6 up to the zero elevation or to elevation 4180 feet in this case. Starting with elevation 4190 upwards, the remaining values of Column 6 are found by multiplying the figures in Column 5 by K or for example at elevation 4220, this amounts to 1.280 x 212 = 271.

Step 11

The average end-area formula is used to compute the sediment volumes of Column 7. Thus between elevations 4230 and 4240 this volume would be (269 + 259) 10/2 = 2,640 acre-feet. It is noted that the areas in Column 6 are used in this computation. A summation of the values in Column 7 shows a total volume of 24,500 acre-feet which is well within the practical limits of the estimated total sediment inflow, 24,600 acre-feet.

Step 12

Column 8 is the accumulated values of Column 7.

Step 13

Column 9 equals Column 2 minus Column 6.

Step 14

The values of Column 10 are found by subtracting Column 8 from Column 3.

Step 15

A plot is made of the results as shown in Figure 8. The data from Columns 1, 2, 3, 9, and 10 are used to plot this graph.

CONCLUSIONS

The foregoing revised procedure is another technical improvement of the existing method of computing the sediment distribution in large reservoirs. One of the major results of the procedure is that it eliminates the cut and try method of determining the zero depth.

This revised procedure is recommended for use in future sediment distribution computations. The example presented in this report can be used as a guide in making such computations. It is also particularly useful in preliminary studies on determining directly the elevation of sediment deposited at the dam for any given time and volume of sediment.

LITERATURE CITED

Borland, W. M. and Miller, C. R., 1960, Distribution of sediment in large reservoirs: Transactions, American Society of Civil Engineers, v. 125, p. 166-180.

Moody, W. T., 1962, Determination of the maximum elevation of complete sedimentation in a reservoir: Memorandum, U. S. Bureau of Reclamation, 4 p., 5 figs., 2 tables (unpublished).

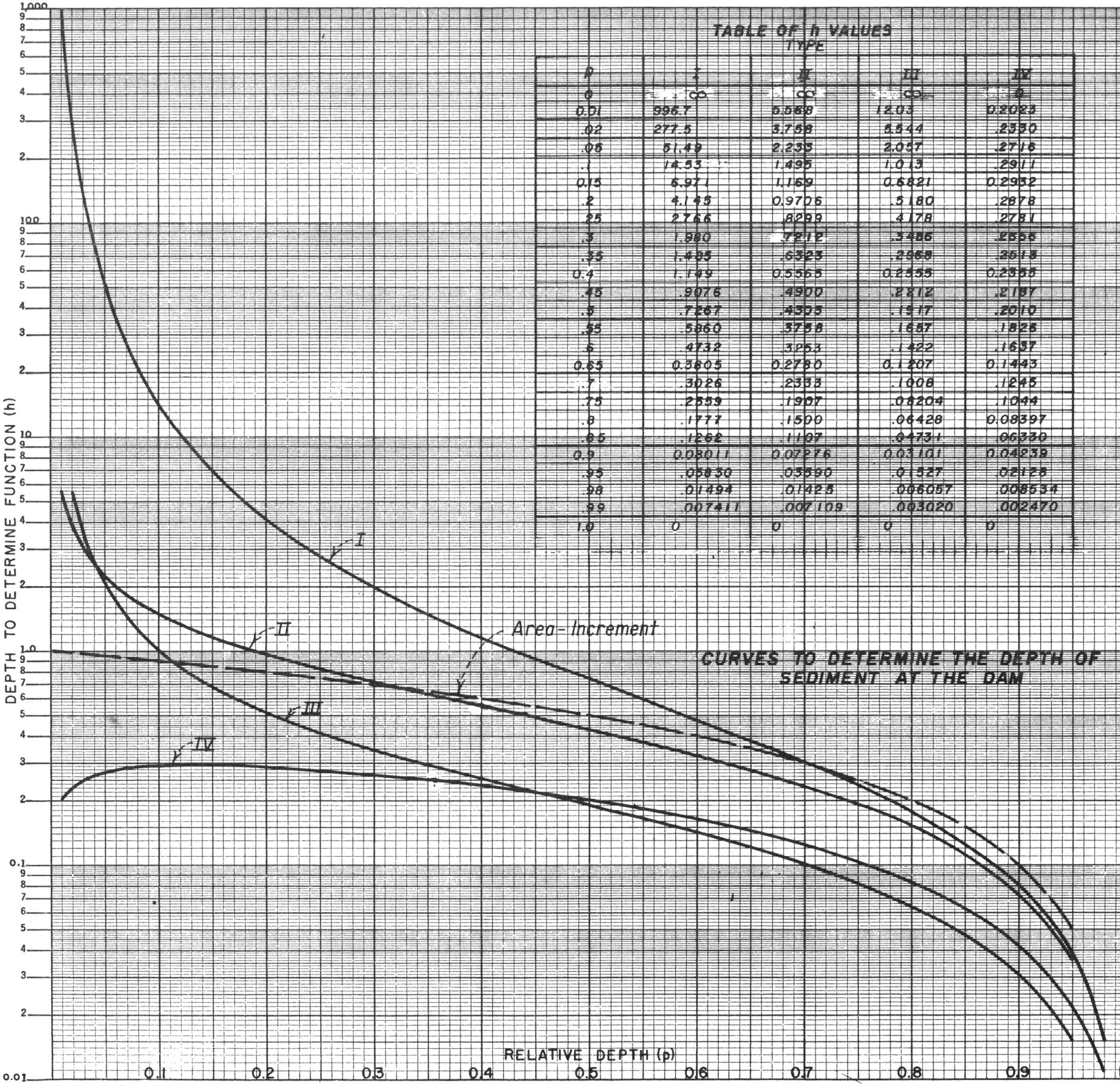


TABLE OF h VALUES

p	Type I	Type II	Type III	Type IV
0	∞	∞	∞	0
0.01	996.7	5.568	12.03	0.2023
.02	277.5	3.758	5.544	.2330
.05	51.49	2.235	2.057	.2716
.1	14.53	1.495	1.013	.2911
0.15	6.971	1.169	0.6821	0.2932
.2	4.145	0.9706	.5180	.2878
.25	2.766	.8299	.4178	.2781
.3	1.980	.7212	.3486	.2656
.35	1.485	.6323	.2968	.2513
0.4	1.149	0.5565	0.2555	0.2355
.45	.9076	.4900	.2212	.2187
.5	.7267	.4305	.1917	.2010
.55	.5860	.3758	.1657	.1826
.6	.4732	.3253	.1422	.1637
0.65	0.3805	0.2780	0.1207	0.1443
.7	.3026	.2333	.1008	.1245
.75	.2359	.1907	.08204	.1044
.8	.1777	.1500	.06428	0.08397
.85	.1262	.1107	.04731	.06330
0.9	0.08011	0.07276	0.03101	0.04239
.95	.03830	.03590	.01527	.02128
.98	.01494	.01425	.006057	.008534
.99	.007411	.007109	.003020	.002470
1.0	0	0	0	0

FIGURE 1

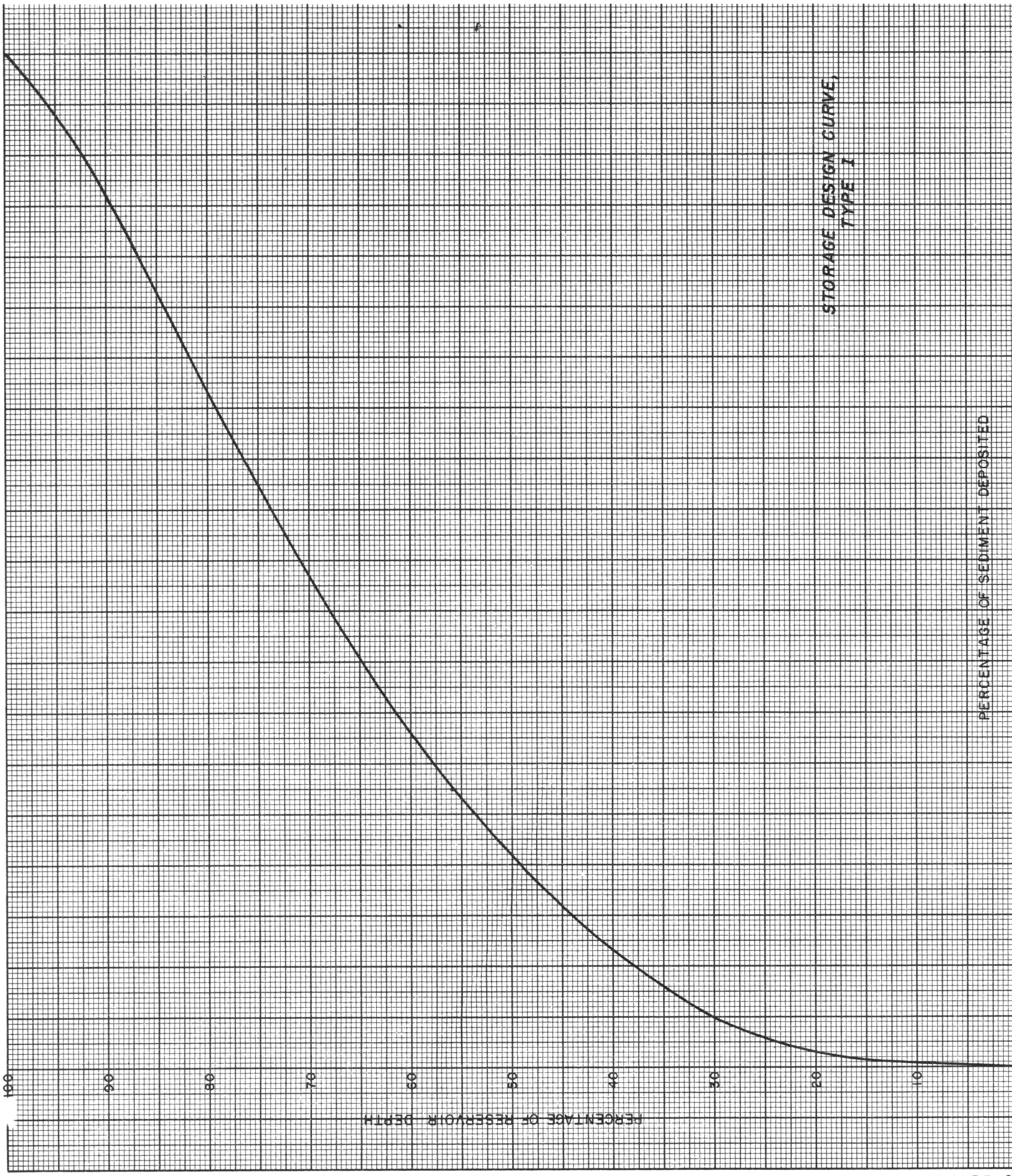

STORAGE DESIGN CURVE, TYPE I
PERCENTAGE OF SEDIMENT DEPOSITED
PERCENTAGE OF RESERVOIR DEPTH
0
10
20
30
40
50
60
70
80
90
100

FIGURE 2

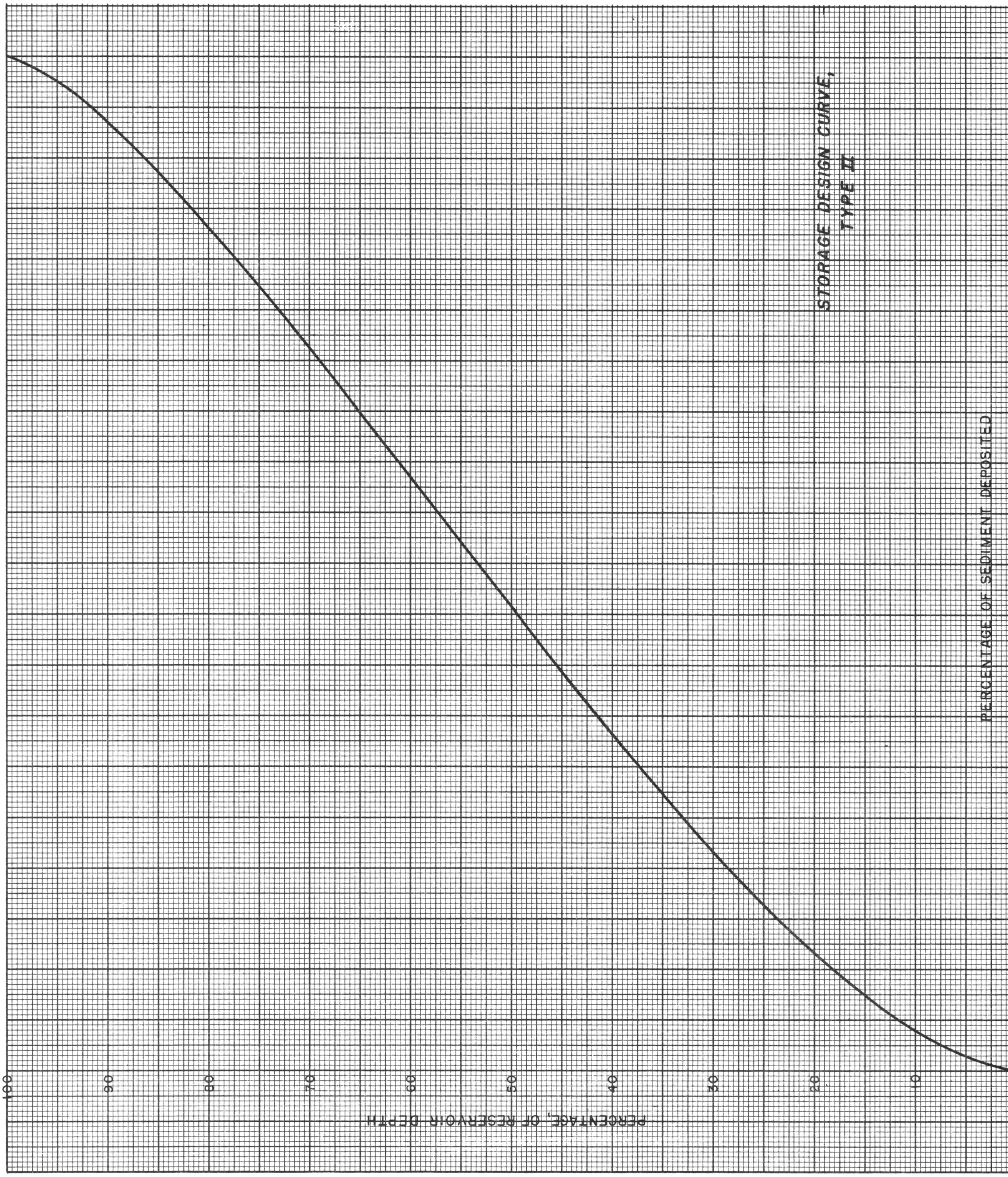
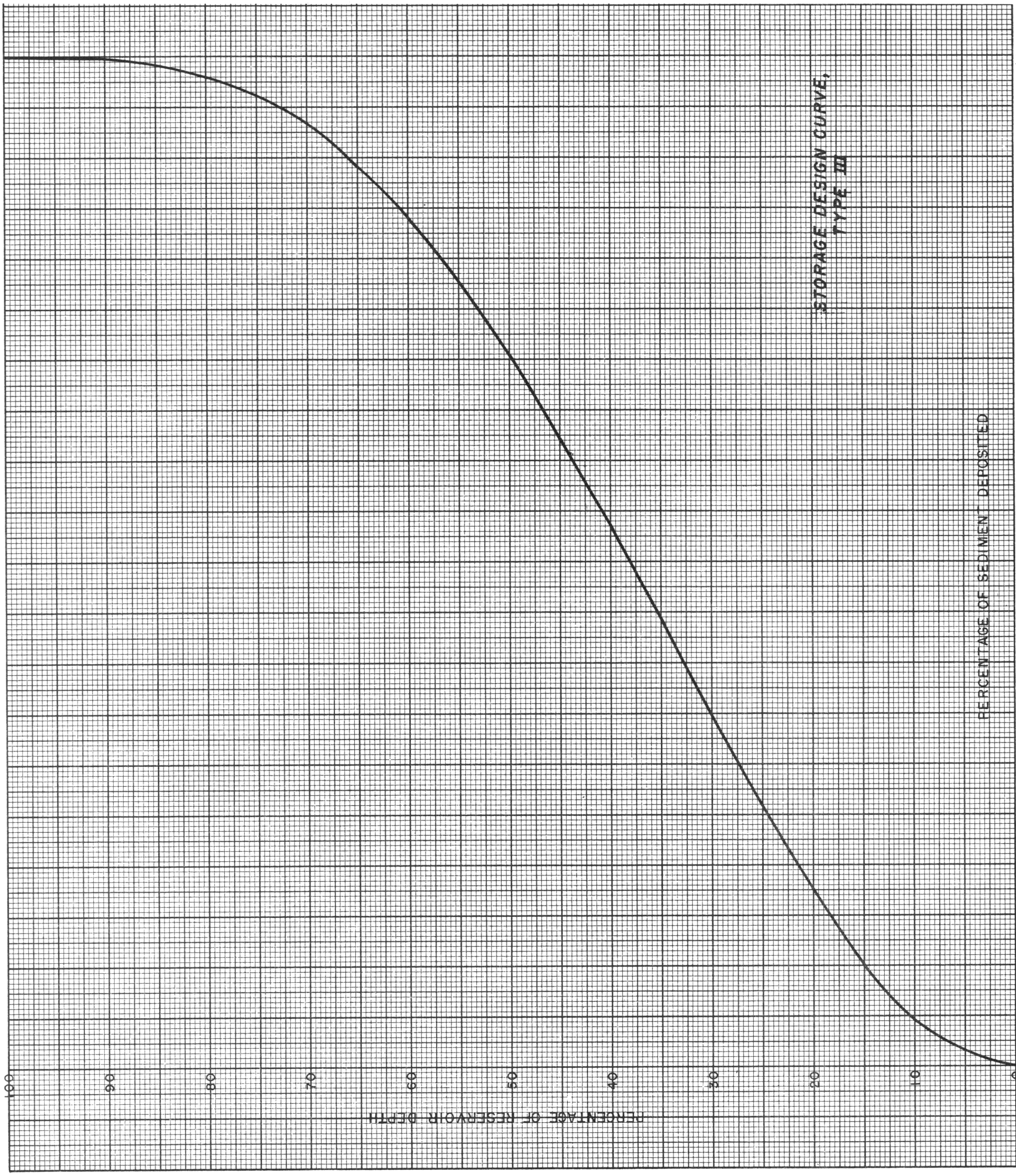

STORAGE DESIGN CURVE, TYPE II
PERCENTAGE OF SEDIMENT DEPOSITED
PERCENTAGE OF RESERVOIR DEPTH
0
10
20
30
40
50
60
70
80
90
100

FIGURE 3



STORAGE DESIGN CURVE,
TYPE III
PERCENTAGE OF SEDIMENT DEPOSITED
PERCENTAGE OF RESERVOIR DEPTH
0
10
20
30
40
50
60
70
80
90
100

FIGURE 4

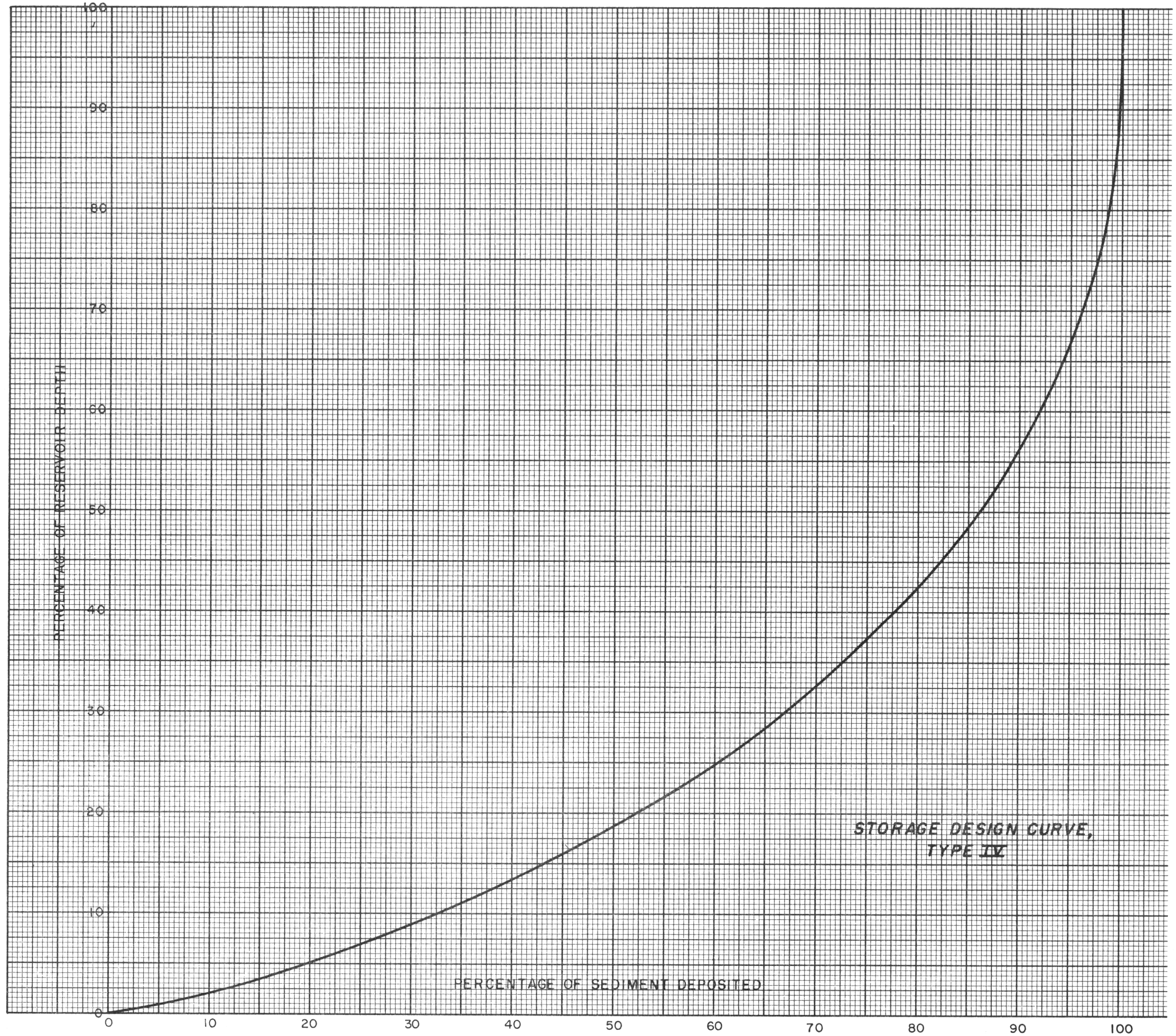
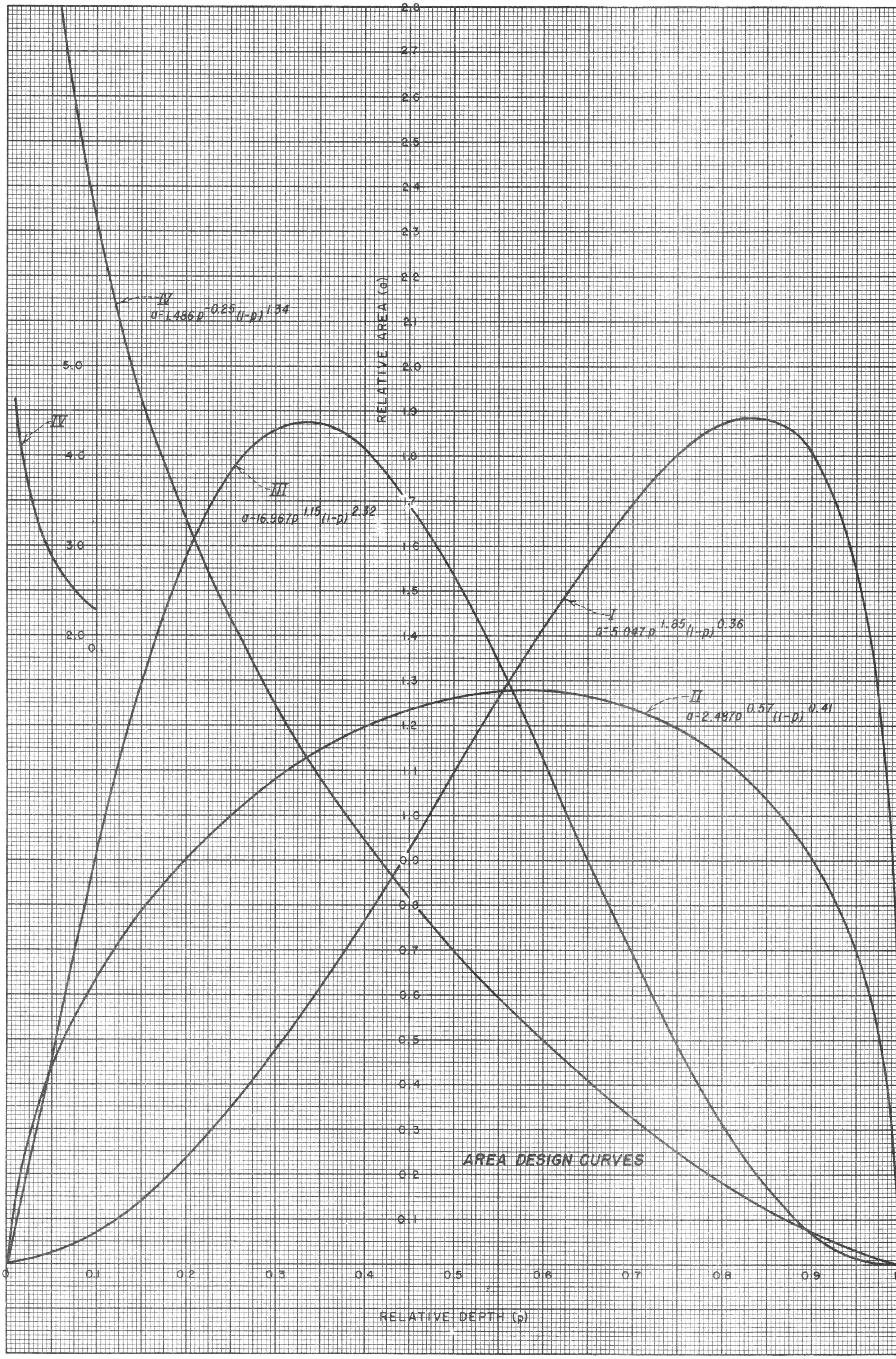

STORAGE DESIGN CURVE,
TYPE IV
PERCENTAGE OF RESERVOIR DEPTH
PERCENTAGE OF SEDIMENT DEPOSITED
0
10
20
30
40
50
60
70
80
90
100

FIGURE 5



AREA DESIGN CURVES
RELATIVE AREA (a)
RELATIVE DEPTH (p)
I
a=5.047 p^1.85 (1-p)^0.36
II
a=2.487p^0.57 (1-p)^0.41
III
a=16.967p^1.15 (1-p)^2.32
IV
a=1.486 p^-0.25 (1-p)^1.34
IV

FIGURE 6

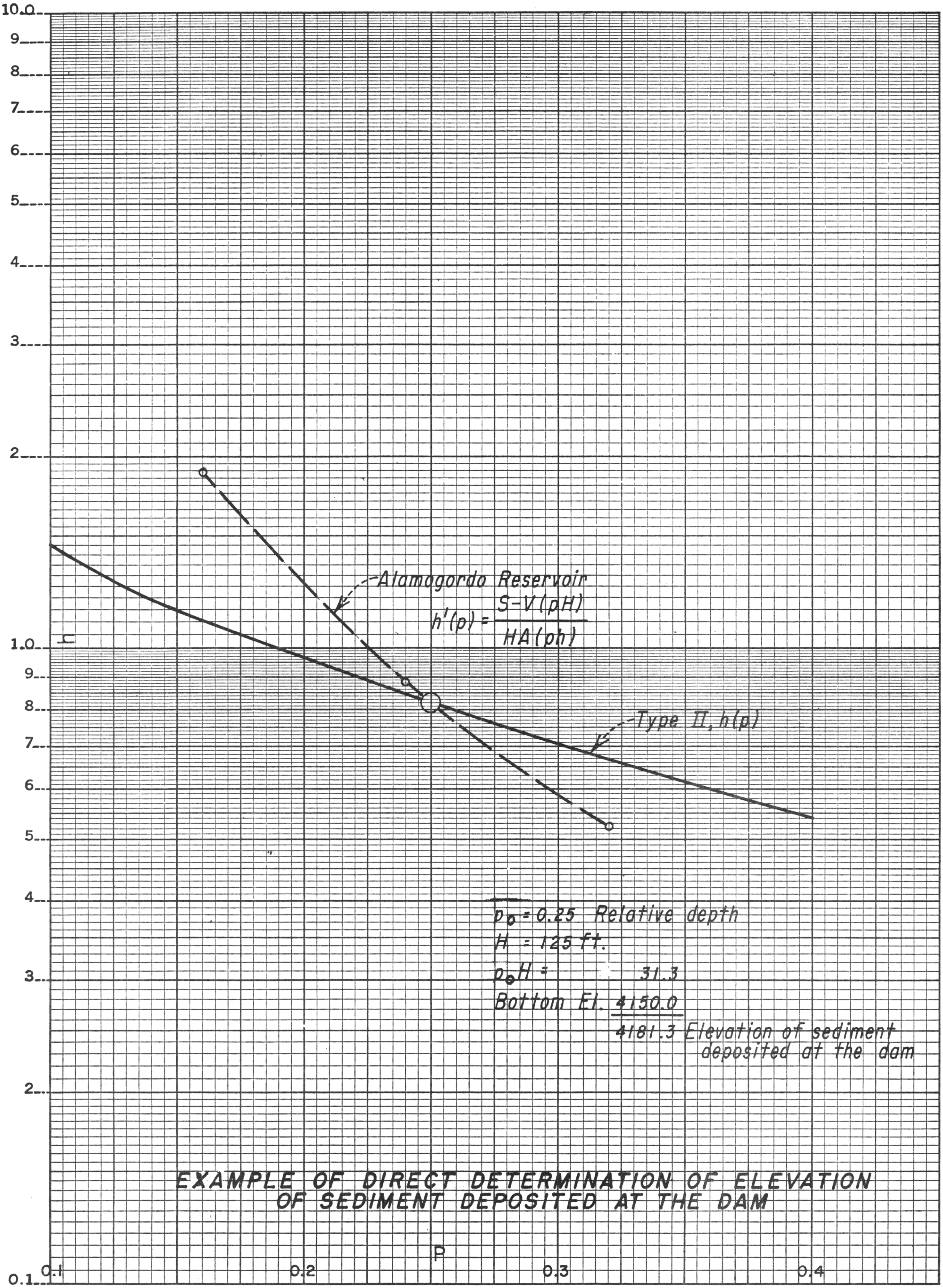

10.0
9
8
7
6
5
4
3
2
1.0
9
8
7
6
5
4
3
2
0.1
h
Alamogordo Reservoir
h'(p) = S-V(pH) / HA(ph)
Type II, h(p)
p₀ = 0.25 Relative depth
H = 125 ft.
p₀H = 31.3
Bottom El. 4150.0
4181.3 Elevation of sediment deposited at the dam
EXAMPLE OF DIRECT DETERMINATION OF ELEVATION OF SEDIMENT DEPOSITED AT THE DAM
P
0.1
0.2
0.3
0.4

FIGURE 7

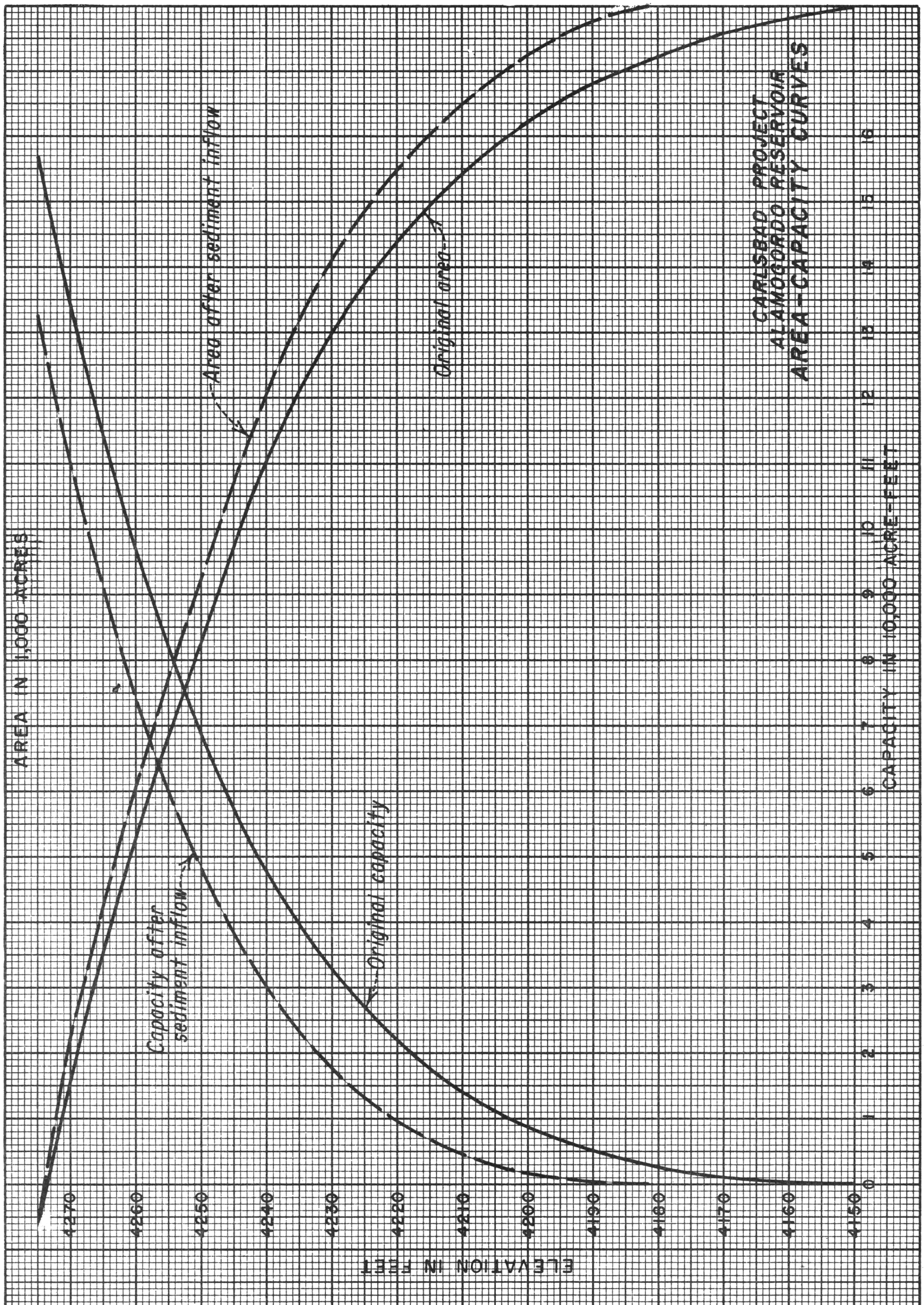

CARLSBAD PROJECT
ALAMOGORDO RESERVOIR
AREA-CAPACITY CURVES
AREA IN 1,000 ACRES
0
1
2
3
4
CAPACITY IN 10,000 ACRE-FEET
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
ELEVATION IN FEET
4270
4260
4250
4240
4230
4220
4210
4200
4190
4180
4170
4160
4150
Area after sediment inflow
Original area
Original capacity
Capacity after sediment inflow

FIGURE 8

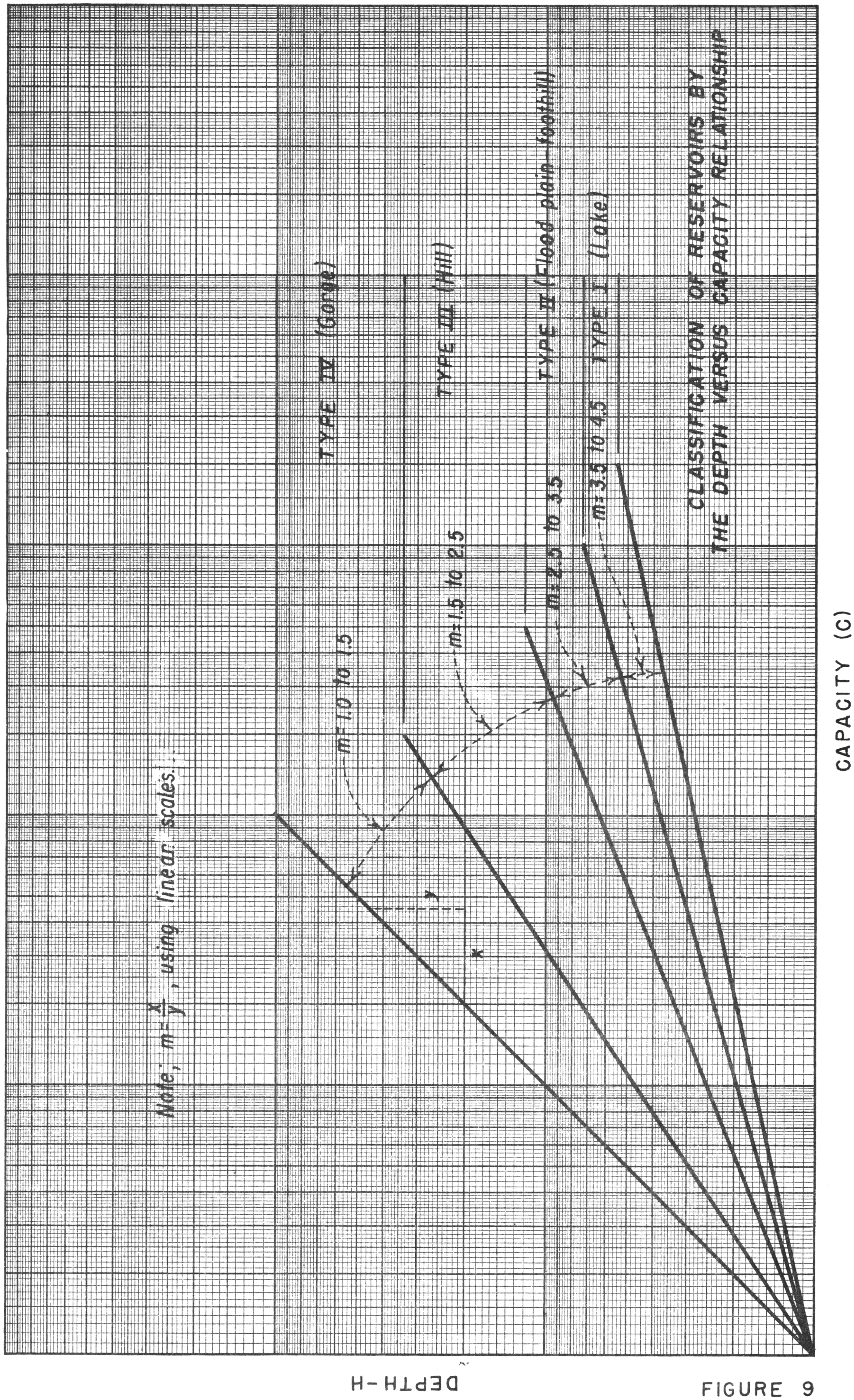

Note; m = x/y, using linear scales.
TYPE IV (Gorge)
TYPE III (Hill)
TYPE II (Flood plain-foothill)
TYPE I (Lake)
m=1.0 to 1.5
m=1.5 to 2.5
m=2.5 to 3.5
m=3.5 to 4.5
x
y
CLASSIFICATION OF RESERVOIRS BY
THE DEPTH VERSUS CAPACITY RELATIONSHIP
CAPACITY (C)
DEPTH-H

FIGURE 9

SEDIMENT DISPOSITION COMPUTATIONS

Table 1

(Empirical Area-Reduction Method)

Reservoir ALAMOGORDO Project CARLSBAD

Total Sediment Inflow 24,600 acre feet Computed by JML Date 3-3-62

① Elevation (feet)	② Original Area (acres)	③ Original Capacity (acre-feet)	④ Relative Depth	⑤ A_p Type II	⑥ Sediment Area (acres)	⑦ Sediment Volume (acre-feet)	⑧ Accumulated Sediment Volume (acre-feet)	⑨ Revised Area (acres)	⑩ Revised Capacity (acre-feet)
4275	4,650	156,750	1.00	0	0		24,500	4,650	132,250
						340			
4270	4,100	133,500	.96	.64	136		24,160	3,964	109,340
						1,710			
4260	3,200	97,000	.88	.97	206		22,450	2,994	74,550
						2,225			
4250	2,450	68,750	.80	1.129	239		20,225	2,211	48,525
						2,490			
4240	1,750	47,750	.72	1.220	259		17,735	1,491	30,015
						2,640			
4230	1,250	32,750	.64	1.269	269		15,095	981	17,655
						2,700			
4220	900	22,000	.56	1.280	271		12,395	629	9,605
						2,680			
4210	650	14,250	.48	1.250	265		9,715	385	4,535
						2,595			
4200	450	8,750	.40	1.196	254		7,120	196	1,630
						2,445			
4190	300	5,000	.32	1.108	235		4,675	65	325
						2,175			
4180	200	2,500	.24	.98	200		2,500	0	0
						1,500			
4170	100	1,000	.16	.81	100		1,000	0	0
						750			
4160	50	250	.08	.57	50		250	0	0
						250			
4150	0	0	0	0	0		0	0	0

SUPPLEMENT

4181.3	212		.25	1.00	K = 212 ÷ 1.00 = 212

Table 2

DIRECT DETERMINATION OF ELEVATION OF SEDIMENT DEPOSITED AT THE DAM

(Empirical Area-Reduction Method)

Reservoir ALAMOGORDO   Project CARLSBAD

S = 24,600 acre-feet   H = 125 feet

① ELEV. (FT)	② p	③ V (pH)	④ S-V(pH)	⑤ HA(pH)	⑥ h'(p)
4150	0	0	24,600	--	--
4160	0.08	250	24,350	6,250	3.9
4170	0.16	1,000	23,600	12,500	1.89
4180	0.24	2,500	22,100	25,000	0.884
4190	0.32	5,000	19,600	37,500	0.523

p_o = 0.25

$p_o H$ = 31.3

Bottom elevation = 4150.0

Elevation of sediment deposited at dam = 4181.3

NOTATION OF SYMBOLS

p = relative depth of reservoir.

V(pH) = reservoir capacity in acre-feet at a given elevation.

S = total sediment inflow in acre-feet.

H = height of dam in feet.

A(pH) = reservoir area in acres at a given elevation.

h'(p) = a function of the reservoir and its anticipated sediment storage expressed as follows:

$$h'(p) = \frac{S - V(pH)}{HA(pH)}$$

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