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APPLICATION OF WAVELET ANALYSIS FOR MONITORING THE HYDROLOGIC EFFECTS OF DAM OPERATION: GLEN CANYON DAM AND THE COLORADO RIVER AT LEES FERRY, ARIZONA

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ABSTRACT

Wavelet analysis is a powerful tool with which to analyse the hydrologic effects of dam construction and operation on river systems. Using continuous records of instantaneous discharge from the Lees Ferry gauging station and records of daily mean discharge from upstream tributaries, we conducted wavelet analyses of the hydrologic structure of the Colorado River in Grand Canyon. The wavelet power spectrum (WPS) of daily mean discharge provided a highly compressed and integrative picture of the post-dam elimination of pronounced annual and sub-annual flow features. The WPS of the continuous record showed the influence of diurnal and weekly power generation cycles, shifts in discharge management, and the 1996 experimental flood in the post-dam period. Normalization of the WPS by local wavelet spectra revealed the fine structure of modulation in discharge scale and amplitude and provides an extremely efficient tool with which to assess the relationships among hydrologic cycles and ecological and geomorphic systems. We extended our analysis to sections of the Snake River and showed how wavelet analysis can be used as a data mining technique. The wavelet approach is an especially promising tool with which to assess dam operation in less well-studied regions and to evaluate management attempts to reconstruct desired flow characteristics. Copyright © 2005 John Wiley & Sons, Ltd.

KEY WORDS: Grand Canyon; wavelets; dams; discharge; Lees Ferry; Colorado River; Glen Canyon Dam

INTRODUCTION

The construction of dams for water supply, irrigation, flood control, and hydroelectric power generation in the western United States has altered dramatically many aspects of river function including: sediment transport (Andrews, 1986; Topping *et al.*, 2000, 2003); channel geomorphology (Williams, 1978; Williams and Wolman, 1986; Everitt, 1993; Grams and Schmidt, 2002); riparian vegetation (Turner and Karpiscak, 1980; Johnson, 1992; Stromberg, 2001; Zamora-Arroyo *et al.*, 2001; Shafroth *et al.*, 2002; Cooper *et al.*, 2003); aquatic ecosystems (Minckley and Deacon, 1991; Power *et al.*, 1996; Robinson and Childs, 2001; Valdez *et al.*, 2001); thermal regimes (Ward and Stanford, 1995); and, most obviously, streamflow regime (Dynesius and Nilsson, 1994; Graf, 1999; Poff and Hart, 2002; Olden and Poff, 2003; Topping *et al.*, 2003). There are many aspects of this regime (Poff *et al.*, 1997), including the existence of cyclic phenomena characterized by variation in periodicities and amplitudes. Daily cycles of melting induce afternoon daily maxima on glacial melt water streams, and desert streams experience daily maxima at night when evapotranspiration is least. Annual cycles of high and low flow occur with regular periodicity in snowmelt and Mediterranean climate streams. Decadal-scale wet and dry cycles are known worldwide. Many species of riparian and aquatic ecosystems exploit these cycles, such as the spawning of some fish or the germination and distribution of some riparian trees and shrubs.

Dams have the potential to disrupt these cycles or to create new cycles unrelated to the natural hydrologic regime. In the case of the Colorado River, dam releases are adjusted to match daily cycles of high and

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low hydroelectric power demand, weekly cycles of low weekend power usage, and seasonal cycles related to the needs of electric heating, air conditioning, or irrigation.

Researchers seeking to quantify the effects of dams typically employ metrics such as flood frequency analysis, flow duration curves, statistical analysis of daily, seasonal, and annual discharge patterns, and more recently, computational analysis of a suite of hydrologic parameters (Richter *et al.*, 1996). These methods, in use by a large management and scientific community, rely on *a priori* assignment of the time intervals of analysis and do a relatively poor job of evaluating changes in cyclic phenomena. Statistical tests based on pre-assigned time intervals are limited by the time frame selection and characterize cyclic phenomena by means, variance, periods, or amplitudes of each time interval. Since pre-dam conditions often include decadal-scale wet and dry periods and post-dam periods include months to years of changing dam operating rules, it is preferable that statistical tests that evaluate the effects of dams on cyclic phenomena accommodate a continuous range of time periods of analysis.

Spectral analysis, in particular wavelet analysis (Daubechies, 1992; Farge, 1992; Liu, 1994; Kumar and Foufoula-Georgiou, 1997; Mallat, 1999), provides alternative methods wherein variations in streamflow can be analysed without the necessity of pre-assigning time frames. In particular, wavelets can be used to localize simultaneously modulations in the scale (inverse of frequency, analogous to period) and amplitude of streamflow, an approach that is not possible with traditional hydrograph analysis. Wavelet background and equations are provided in the Appendix.

Wavelets have been employed in several hydrologic analyses including: detection of changes in streamflow variance (Cahill, 2002); simulation of streamflow (Bayazit and Aksoy, 2001; Bayazit *et al.*, 2001); identification of climatic impacts on Oregon streamflow (Bradshaw and McIntosh, 1994); characterization of streamflow patterns in remote tropical (Gaucherel, 2002) and glacial (Lafreneire and Sharp, 2003) landscapes; analysis of Nile flood patterns (Jiang *et al.*, 2002); differentiating between natural and anthropogenic influences on streamflow (Nakken, 1999); and identification of hydrologic regions (Smith *et al.*, 1998; Saco and Kumar, 2000). To our knowledge, no research has yet employed wavelets to analyse hydrologic changes caused by dams with the high temporal resolution data evaluated here.

In the western United States, the creation and operation of dams is of great interest to a large political, recreational, management, and scientific community. Since the March 1963 closure of the Glen Canyon Dam, which created Lake Powell reservoir, the hydrology of the Colorado River in Grand Canyon has been altered greatly, leading to ongoing debate about downstream river management for the past 20 years (Schmidt *et al.*, 1998).

Four conditions led us to focus attention on analysis of the effects of Glen Canyon Dam on the Colorado River: intense public interest in dam management and dams of the Colorado River; good records of the history of dam operation; accepted tools for wavelet analysis; and the recent development of a continuous record of instantaneous discharge for the Colorado River at Lees Ferry, Arizona, from 1921 to 2000 (Topping *et al.*, 2003). Gauging began at Lees Ferry because it is the first location downstream from the Green, upper Colorado, and San Juan Rivers where Colorado River crossings were possible and automobile access was possible in the 1920s. Additionally, Lees Ferry was near the 1922 location of the Colorado River Compact division of the Upper and Lower Basins. The Lees Ferry record is therefore of great historical, geomorphic, and hydrological interest.

The gauging station at Lees Ferry, located 25 km downstream from Glen Canyon Dam (Figure 1), measures the essential attributes of dam releases while upstream gauges provide a good record of the quasi-natural inflow to the reservoir (between 10 and 20% of upstream flow has been depleted by agricultural diversion). See Topping *et al.* (2003) for extensive background on the history and operation of the Lees Ferry gauging station. Given these conditions, our goal was to conduct wavelet analyses of daily mean and continuous discharge at Lees Ferry. We first present the natural flow regime of the Colorado River and a traditional methods section. The remainder of the paper is structured as a series of research questions and answers that geomorphologists, hydrologists, and ecologists are likely to raise prior to adopting the wavelet technique.

THE NATURAL FLOW REGIME OF THE COLORADO RIVER

Prior to the construction of Glen Canyon Dam, annual snowmelt floods peaking in May or June and winter low flows were the defining natural cycle of streamflow at Lees Ferry. The rising limb of the annual peak varied between gradual and precipitous depending on the relative timing and duration of snowmelt in the Rocky

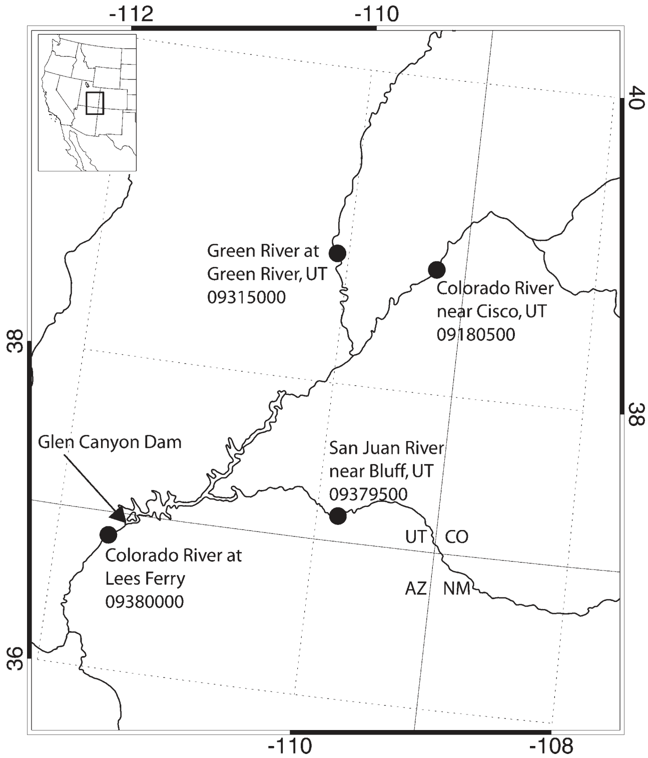

Figure 1. Study region. Numbers indicate USGS gauging station identification numbers

Mountains and the surrounding low elevation terrain. The magnitude of the annual flood was generally related to the magnitude of the Rocky Mountain snow pack and the duration of the melt season. Between August and October of each year, the summer thunderstorm season generated short duration floods that typically were less than half the magnitude of the snowmelt flood. This highly variable precipitation, driven by both the monsoon and dissipating tropical cyclones, determined the number of late summer and early autumn floods. Shorter period natural cycles have not been described but multi-year periods of wet and dry have been measured using tree ring surrogates (Meko *et al.*, 1995).

Total annual flow in the Colorado River does not change significantly downstream from the confluence of the Colorado and San Juan Rivers. For the pre-dam period, the combined daily mean discharge of the Colorado River near Cisco, UT (USGS station number 09180500), the Green River at Green River, UT (USGS station number 09315000), and the San Juan River near Bluff, UT (USGS station number 09379500) (Figure 1) is virtually identical to that measured at Lees Ferry ($r^2 = 0.99$ for correlation of daily mean discharge, slope $= 1.01$). Thus, it is possible to compare the attributes of inflow to Lake Powell with attributes of dam release by comparing the combined upstream gauging records with those for Lees Ferry.

METHODS

Data

We based our analysis of the Lees Ferry record on data produced in Topping *et al.* (2003), who digitized the original stage records, applied appropriate rating data, implemented extensive quality control checks, and produced sub-daily discharge values at time intervals ranging from several minutes to several hours. Topping *et al.* (2003) then analysed the continuous record of instantaneous discharge at Lees Ferry for flow duration, sub-daily discharge variability, and flood frequency, and investigated the implications of natural and dam-induced changes in discharge for sand transport and storage in Grand Canyon National Park. All of Topping *et al.*'s analyses required

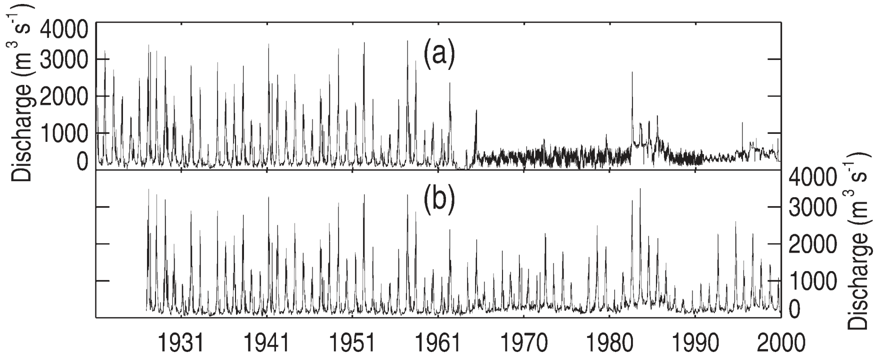

Figure 2. Daily mean discharge at Lees Ferry: (a) from Topping *et al.* (2003) and from the combined upstream tributaries (b) shown in Figure 1

the *a priori* assignment of the time intervals of analysis. This study builds on the results of Topping *et al.* (2003) because wavelet analysis does not require this imposition of time intervals and therefore allows recognition of natural and dam-induced cyclic phenomena occurring at any time and at any scale.

Following retrieval of the continuous record of instantaneous discharge, we concatenated the individual files, computed Julian time for each record, and calculated daily mean discharge for Lees Ferry (Figure 2a) and the upstream tributaries (Figure 2b from stations shown in Figure 1). For additional analysis (see below), we obtained daily mean discharge data for the Snake River near Moran, Wyoming (USGS station 13011000, dammed but operated with a very different approach from Glen Canyon Dam) and at Hell's Canyon (USGS stations 13290000 for October 1925 to March 1958, 13290200 for March 1958 to December 1966 and 13290450 for 1967 to 2002). Although the Hell's Canyon gauging stations are not identical, correlations during periods of overlap are very high.

Because wavelet analysis should be executed with a time series at evenly spaced intervals and the Lees Ferry record consists of highly variable intervals, we created a time series with one-hour intervals by linear interpolation of the continuous record (Figure 3). In very limited cases where more than half a day's data were missing, we did not calculate hourly values (with no detectable effect on results).

Spectral analysis

Our purpose in this research was to reveal the spectral structure of the Lees Ferry record as influenced by natural and anthropogenic effects, not to fully describe the wavelet technique, which has been done elsewhere (Torrence and Compo, 1998). Nonetheless, we provide an abbreviated explanation of the wavelet technique in the Appendix.

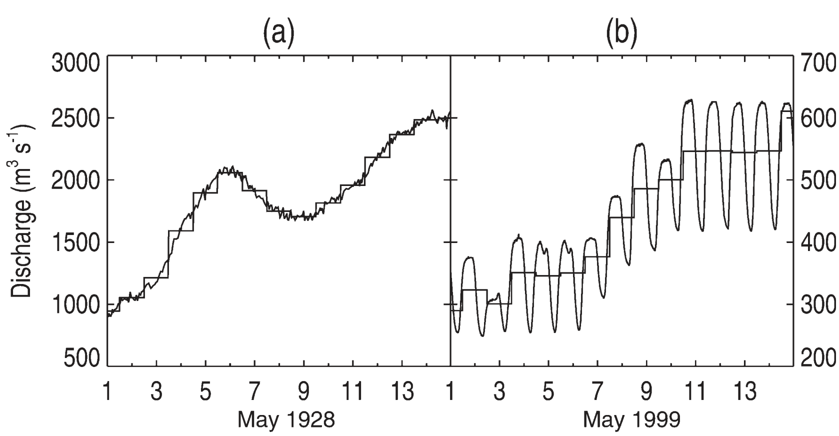

Figure 3. Hourly discharge for two-week subsets of the pre-dam (a, May 1928) and post-dam (b, May 1999) periods. Stepped lines show daily mean discharge

We first calculated the Fourier power spectrum (FPS, normalized by variance) of the daily mean discharge data for pre- and post-dam periods. We then conducted several wavelet analyses (see Appendix for methodological details). For daily mean discharge at Lees Ferry, these included: (1) wavelet power spectrum (WPS) with significance levels based on a random noise process; (2) a statistical approach summarizing pre- versus post-dam conditions; and (3) WPS normalized by the pre-dam local wavelet spectrum. These analyses showed the scale and amplitude modulation of discharge caused by natural variability in the pre-dam period and by dam management combined with residual natural variability in the post-dam period.

Three additional wavelet techniques can provide a further refinement of the overall WPS. First, a subset of scales may be extracted and averaged through time. This provides a graphical illustration of the modulation of a particular range of scales through time and is termed the scale-averaged WPS. Second, by averaging each scale over all time periods, a global wavelet spectrum (GWS) is obtained: this is analogous to FPS. Third, a local wavelet spectrum (LWS) may be calculated for periods of interest. The LWS, like the GWS, is a time-averaged analysis, but provides an averaging over a specific interval, not the entire record. See the Appendix for details.

Using these techniques, we identified a feature in the post-dam Lees Ferry record that could have been induced by either dam management or natural cycles. For this range of scales, we extracted the scale-averaged WPS for records at Lees Ferry, the aggregated upstream stations (Figure 2b), and at the Snake River near Moran, Wyoming, and calculated significance levels based on the records' global wavelet spectrum (GWS).

Next, we compared pre- and post-dam conditions at Lees Ferry using the continuous record of instantaneous discharge at Lees Ferry (Topping *et al.*, 2003) (available at http://www.gcmrc.gov). To do so, we extracted equal length pre-dam (1928 to 1963) and post-dam periods (1964 to 2000) and calculated the WPS and random noise 95% confidence intervals. In an analysis we expect to be of most interest to the management and/or restoration community, we normalized the continuous discharge WPS with the local wavelet spectrum of both the pre-dam and post-dam record. Here, wavelet power that is statistically different than the expected value (pre- or post-dam discharge wavelet power) is displayed: this approach highlights unusual discharge features, i.e. a consistently strong feature in the overall WPS will not appear but a rare event, such as abnormally large floods, will be highlighted.

Finally, to illustrate the potential of the wavelet technique to assess dam operation in less-known systems, we assessed the WPS of the Snake River at Hell's Canyon for modulations in the post-dam operational period. This analysis shows, for a dam system with unknown variations in dam operation, how the wavelet approach can be used as an effective data mining technique to identify modulation in discharge amplitude at specific scales.

RESULTS AND DISCUSSION

Why go to all this trouble? What does wavelet analysis tell us that we can't learn from just looking at the hydrographs?

Wavelet analysis requires no a priori assumption about the timing or length of important processes and provides an easy to interpret image of the amplitude of cycles at all scales and at all times.

Traditional hydrologic analyses, as in the Indicators of Hydrologic Alteration (Richter *et al.*, 1996) or approaches implemented for the Lees Ferry record by Topping *et al.* (2003) require *a priori* assignment of the time intervals of analysis. For these and related techniques, the user must specify the periods for which to assess statistical differences; i.e. a large component of the answer (when did a change in management occur) must be known before the analysis. For dammed rivers with good operational records, it may be possible to identify, prior to analysis, the relevant management actions and consequently the relevant intervals for analysis. For the Colorado River, the creation of Glen Canyon Dam and diurnal discharge cycles in response to power needs are starting points. In cases with a less well-understood record of operation it may not be possible to identify the relevant cycles or timing of events. Wavelet analysis is advantageous because it requires no specification whatsoever of relevant events or cycles and can therefore operate as an efficient data mining technique. Further, the WPS provides, in a single image, a depiction of localized discharge amplitude at all scales, a representation that would otherwise require the production of numerous graphs. In short, we feel that the wavelet approach is exceptionally well suited for analysis of flow regimes in which: (1) the management history is uncertain; (2) the time scale of important cycles is unknown; (3) the depiction of multiple cycles at the same time period is desired.

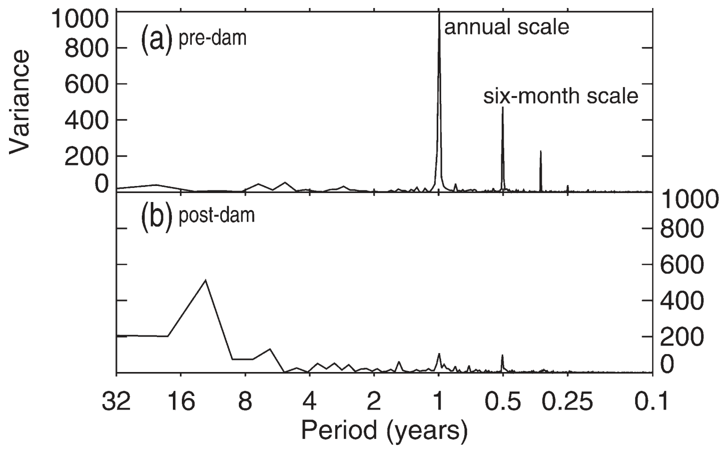

Figure 4. Fourier power spectrum for the pre-dam (a) and post-dam (b) periods calculated from daily mean discharge. The spectrum is normalized by variance

If a spectral analysis is a good idea, why bother with wavelets? Isn't a Fourier analysis good enough?
Fourier analysis does not provide scale and amplitude localization.

The more familiar Fourier analysis is an excellent means of identifying the dominant periodicity in any time series. For the pre-dam period, the annual scale was clearly dominant, with a smaller peak at six months (Figure 4a). In the post-dam period, the annual scale was removed and a cycle with an approximately ten-year period dominated (Figure 4b). In spite of identifying dominant periodicity (analogous to scale in the WPS), the Fourier transform provides no information on modulation in the timing or amplitude of these periodicities. A windowed Fourier transform could provide some information in this regard, but is limited by the choice of the window size, the assumption of decomposability into sinusoidal components, and by a varying number of signal oscillations as a function of frequency.

Does the WPS accurately represent the known history of the Colorado River at Lees Ferry with no 'false positives'?
Yes.

Daily mean discharge is a poor representation of the actual dam release pattern (Figure 3), as demonstrated by Topping *et al.* (2003). Nevertheless, the WPS from daily mean discharge (Plate 1) shows cyclic patterns in normal and dam-induced streamflow that differ greatly from one another and that are consistent with the known history of the Colorado River at Lees Ferry (Table I). The annual cycle of repeated, extended base flows occurring each winter followed by the spring snowmelt flood was the most consistent feature of the pre-dam daily mean discharge record, as indicated by the largest peak in the Fourier power spectrum (Figure 4a) and the horizontal red/brown swath at the one year scale (Plate 1). The extent along the *x*-axis of this red swath indicates that this cyclic pattern was of high amplitude, i.e. a large difference between high and low flow, in most pre-dam years. The power of this cyclicity was weak in drought years such as 1934 and 1955, when the difference between flood peak and base flow was low. The strongest cyclicity existed during years of high runoff, such as 1952, because the amplitude of the cycle was greatest: the flood increased but winter base flows remained low.

The daily mean discharge WPS also exhibited high power at shorter time scales (Plate 1). Periods of high flow at times other than the peak spring snowmelt flood created significant wavelet power at monthly scales in many years. In 1941, for example, unusually strong monsoonal discharge in early autumn (see Plate 1 inset) is apparent at the six-month scale. The strength and duration of peak discharge is visible in the CWT as tips extending vertically to short time scales. In essence, the highest and most consistent wavelet power, corresponding to the strength of the dominant discharge cycle in the Fourier analysis (Figure 4a), was at a scale of one year but was also strong at six-month and shorter scales.

The Lees Ferry record for the pre-dam period is very similar to the upstream pre-dam record (WPS not shown). However, these patterns were completely disrupted in March 1963 when Glen Canyon Dam was closed (Plate 1).

Table I. Timeline of Glen Canyon Dam operations

Timing	Management	Mean	SD	Max.	Min.
13 Mar 1963	Dam closure				
Mar 1963–Apr 1965	Low flow, initial filling of Lake Powell	138	132	817	14
Apr–Jun 1965	Artificial floods to increase volume of water in Lake Mead reservoir and to degrade the channel between Glen Canyon Dam and Lees Ferry	876	423	1707	156
Jul 1965–Jun 1980	'Normal operations': reservoir-filling, high diurnal fluctuations	341	190	868	23
Jun 1980–May 1983	'Normal operations' with annual filling of reservoir	385	187	1286	33
May 1983–Jun 1986	Unusually high runoff; bypass tubes and spillway utilized in May and June of each year	722	329	2755	33
Jun 1986–Jun 1990	'Normal operations' with annual filling of reservoir	389	205	1475	41
Jun 1990–Aug 1991	Test flow period	324	192	835	51
Aug 1991–Jan 1996	Interim operating criteria: no discharge less than 142 $m^3 s^{-1}$; no daytime discharge less than 227 $m^3 s^{-1}$; and no discharge more than 566 $m^3 s^{-1}$ (increased to 708 $m^3 s^{-1}$ in 1996)	336	93	629	148
Jan 1996–Mar 1996	Total volume of water released from dam adjusted to accommodate 1996 controlled flood	425	72	565	277
22 Mar 1996–5 Apr 1996	Controlled flood for seven days; four days of low flow before and after	597	328	1300	230
Apr 1996–Feb 1997	Interim operating criteria resume	417	98	588	187
Oct 1996–present	Record of decision flows	455	137	880	187
Feb 1997	Spill-avoidance management	621	98	766	465
Apr 1998	Emergency exception criteria altered operations	376	89	622	220

SD, standard deviation.

New cyclical flow patterns, generally consistent with the known history of dam operation (Table I), appeared in some years and for a few years, but in general the Lees Ferry record diverged dramatically from the upstream tributaries (not shown). The only years in which the Lees Ferry WPS resembled any aspect of the upstream WPS was during the spring 1965 high flows and between 1983 and 1986 when large Rocky Mountain snowmelt generated catchment-scale floods in each year. In the 'normal operations' period of 1965–1980 no consistent patterns of wavelet power were detectable at less than a ten-year scale. In the period of interim operating criteria between 1991 and 1996, wavelet power was generally insignificant at scales less than six years. The 1996 controlled flood (Webb *et al.*, 1999; Schmidt *et al.*, 2001) did not register any significant aspects of the pre-dam cyclicity (i.e. compare the annual scale in 1996 to the annual scale in the pre-dam period).

Is there some way to obtain a summary of the pre-dam versus post-dam WPS?
Yes.

Although one of the main strengths of the wavelet approach is the ability to localize modulations in scale and amplitude, a summary depiction can also be useful. An effective method is to conduct the following steps: (1) extract the shortest scale from the WPS; (2) conduct a *t*-test for the pre-dam versus post-dam record; (3) repeat this analysis for all scales. This approach, when applied to the daily mean discharge record WPS (Plate 1) succinctly illustrates the main differences in the pre-dam versus post-dam periods (Figure 5). Peak reductions in wavelet power occurred at the annual and six month scales with general reductions from about ten days to eight years. Increases in power were restricted to scales shorter than seven days and longer than three years (possibly related to reservoir residence time and/or filling time).

Is additional information gained by using discharge data with a fine temporal resolution?
Yes.

The WPS of the pre-dam period, when created from the continuous discharge data, is nearly identical to the WPS created using daily mean discharge and we do not present it here. The post-dam WPS created from the continuous (Plate 2) versus the daily mean discharge data (Plate 1), though, are strikingly different: much shorter

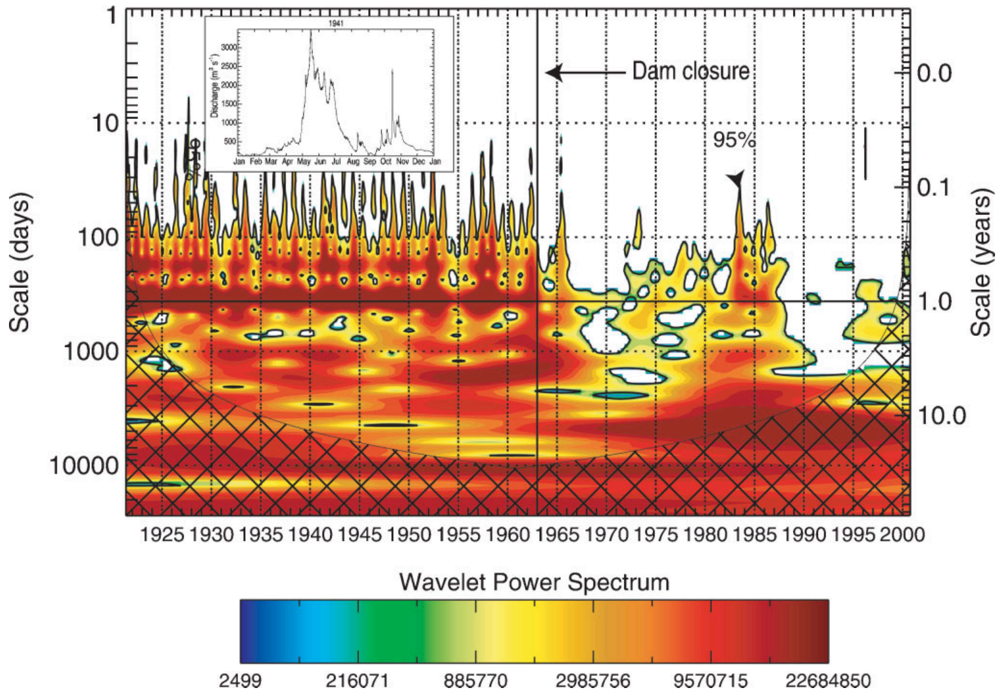

Plate 1. Colorado River discharge at Lees Ferry. Wavelet power spectrum of daily mean discharge with statistically significant results (based on random noise) inside the labelled 95% confidence interval; non-significant wavelet power not shown. Image colours are a representation of the WPS normalized by variance. Solid vertical line in 1963 shows the closure of Glen Canyon Dam. The solid black U-shaped line is the cone of influence (COI), below which edge effects limit confidence in results. The *x*-axis shows time (translation); the logarithmic *y*-axis shows the wavelet scale (dilation). Wavelet descriptions are available in the Appendix

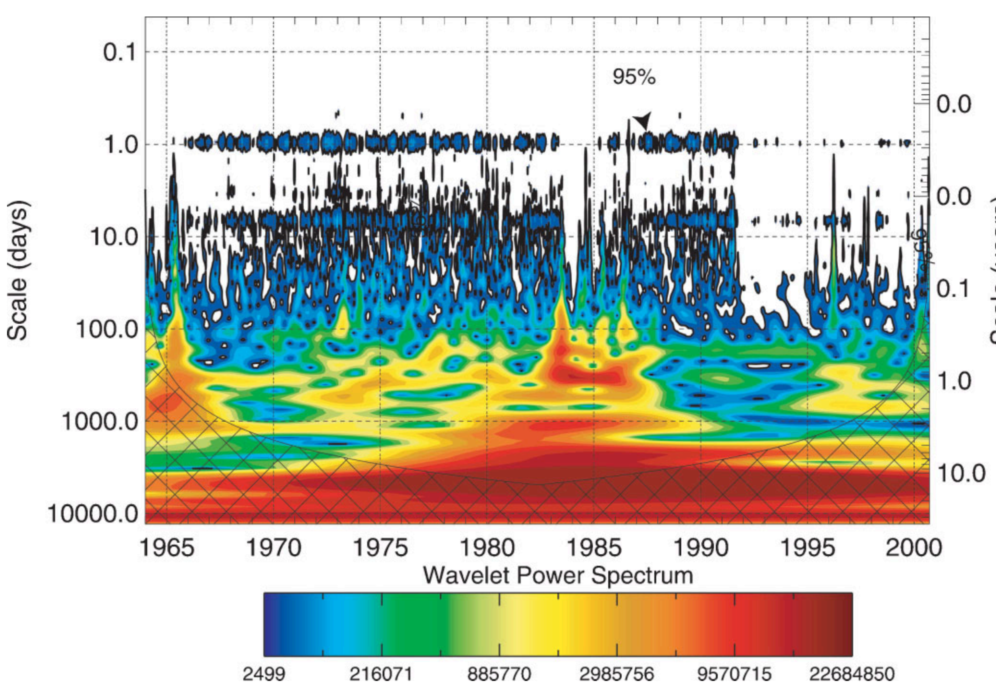

Plate 2. Colorado River wavelet power spectrum of the hourly discharge record in the post-dam period. See Plate 1 caption for details. Wavelet descriptions are available in the Appendix

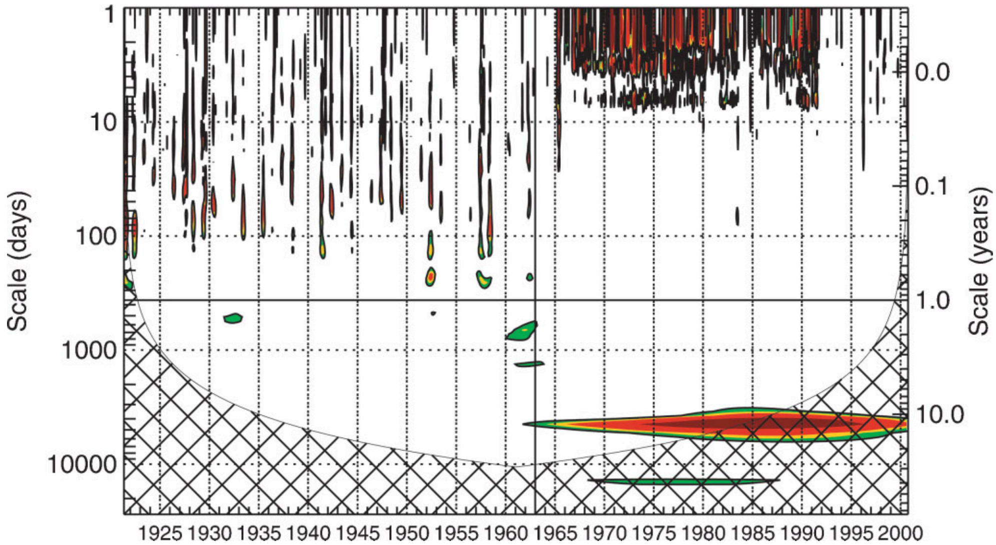

Plate 3. Colorado River daily mean discharge wavelet power spectrum normalized by the 1921–1963 local wavelet spectrum (also used to construct 95% confidence intervals). Solid vertical line shows the closure of Glen Canyon Dam. Coloured contours are drawn at three (green), four (yellow), five (red), and ten (brown) times the expected value. The peak of the 9- to 12-year feature in the mid-1980s, for example, was at least ten times larger than the expected wavelet power at this scale. Details available in Plate 1 caption and the Appendix

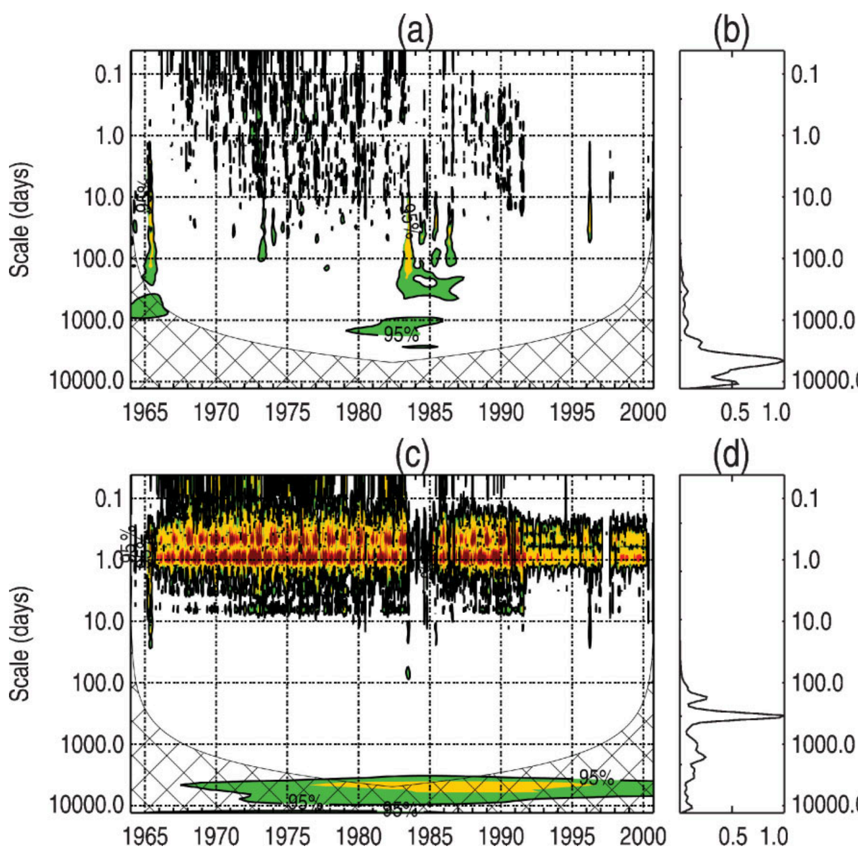

Plate 4. Colorado River post-dam continuous discharge wavelet power spectrum normalized by local wavelet spectra. The spectrum in (a) has been normalized by the post-dam local wavelet spectrum in (b); the spectrum in (c) has been normalized by the pre-dam local wavelet spectrum in (d). Confidence intervals were constructed using local wavelet spectra, which are plotted on a zero to one scale. Coloured contours are drawn at 3 (green), 10 (yellow), 100 (red), and 200 (brown) times the expected value. Details available in Plate 1 caption and the Appendix

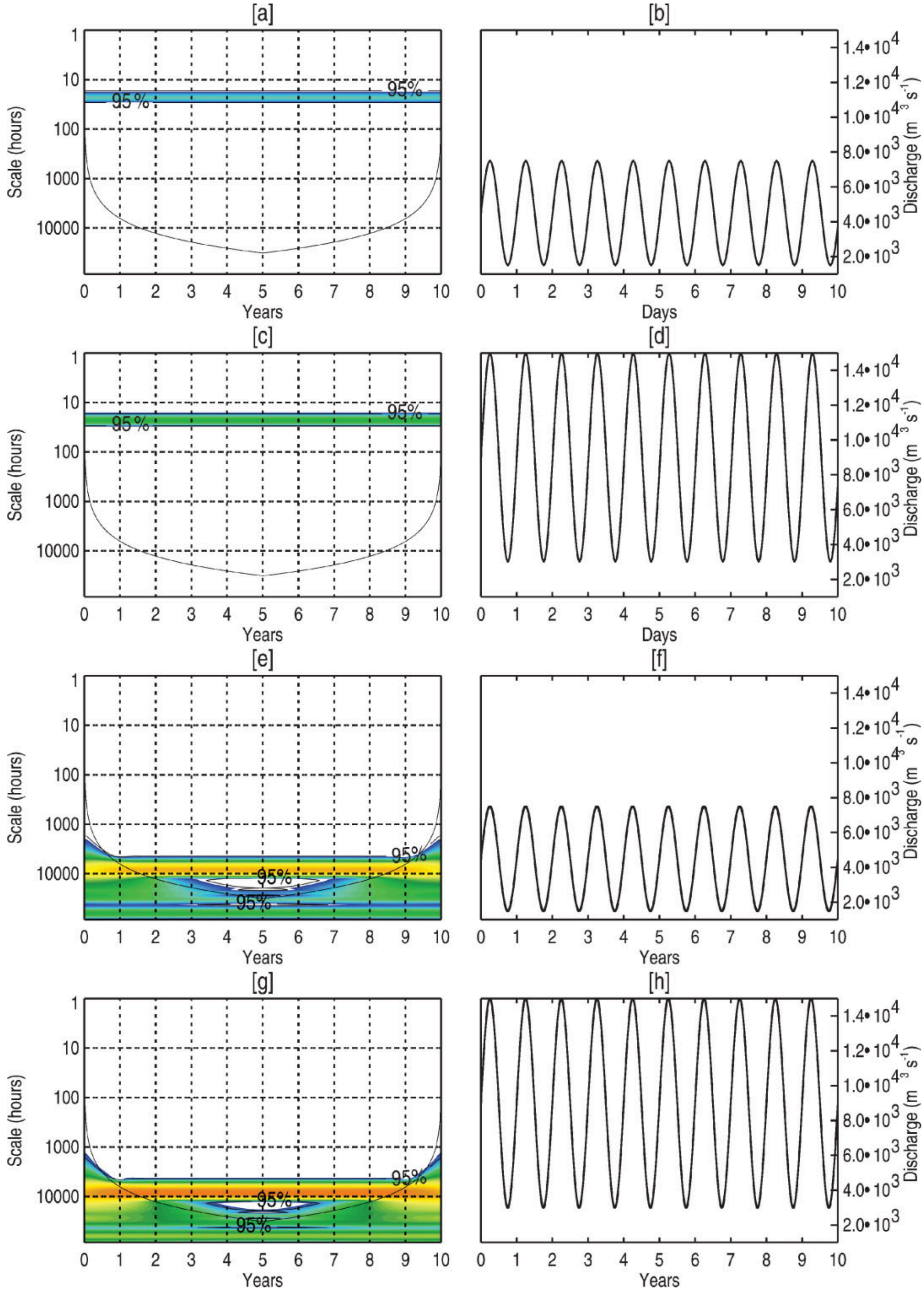

Plate 5. Illustration of relationship between cyclic discharge patterns and the wavelet power spectrum. Each row shows the wavelet power spectrum (left) of a ten-year record of hypothetical hourly discharge (right). Scenarios are discharge with: (1) a purely diurnal cycle at high (a and b) and low average flow and amplitude (c and d) and (2) discharge with a purely annual cycle at high (e and f) and low average flow and amplitude (g and h) flows. Discharge records (b, d, f, h) and wavelet power (a, c, e, g) have consistent *y*-scales. Note that for scenarios with hourly cycles (b and d), *x*-axis is in days. Increasing discharge amplitude, for both diurnal and annual cycles, increases wavelet power, as shown in colour variation. Changing the period of the discharge, i.e. from diurnal to annual, changes the scale at which significant wavelet power occurs. For discharge with an annual cycle, significant power exists at longer than annual scales (e and g). Most power at these scales is subject to edge effects and is below the cone of influence (U-shaped curve)

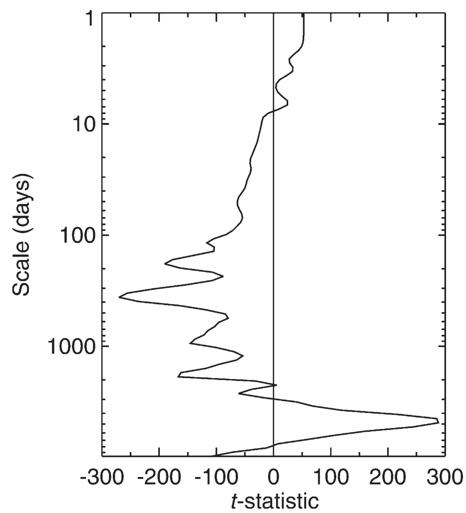

Figure 5. Summary *t*-statistics for comparisons of Lees Ferry pre-dam versus post-dam wavelet power spectrum at every unique scale. All *t*-statistics are significant at the 1% level. Positive values indicate higher post-dam wavelet power

time scale information can be gleaned from the continuous discharge WPS than from the daily mean discharge WPS.

Most obviously, the strong diurnal cycle related to hydroelectric power generation (Figure 3b) was manifested as low, statistically significant, and highly variable wavelet power at the one-day scale. The daily cycle appeared in 1966 and was strongest from around 1970 to 1980. During the high flow events of the mid-1980s the diurnal cycle was eliminated but then returned until August 1991 when dam releases were restricted in an attempt to limit downstream ecosystem effects (National Research Council, 1996). Wavelet power decreased at the one-day scale after 1991 in response to these operational changes and remained low for the duration of the record. Another cycle at the one-week scale, related to the need to lower power production on Sundays, appears in a band of significant wavelet power with variations similar to those for the daily power band.

The operation of Glen Canyon in 'normal' mode (Table I) up to August 1991 was characterized by significant wavelet power from one week up to the cone of influence in most years, indicating the presence of many overlapping scales of operation. After August 1991 and the adoption of interim operating criteria, dam operations greatly reduced nearly all cycles between 1 and 100 days.

The March 1996 controlled flood is marginally more evident in the continuous discharge data (Plate 2) than in the daily mean discharge data. The flood generated significant wavelet power at scales from about 4 to 40 days (Harpman, 1999). Wide-ranging effects on flow regime did in fact occur well beyond the brief seven-day duration of the flood, because flow adjustments six months before and after the flood were made to the regional water storage programme. These effects were difficult to detect in the WPS of daily mean discharge (Plate 1), but are evident in the CWT from continuous data (Plate 2). Nevertheless, the flood seems to have been a minor event bearing only a tangential resemblance to the annual- or sub-annual flow regimes of the pre-dam period.

Do these analyses tell us anything new about dam operations?

Yes.

The preceding analysis confirms that the wavelet approach correctly identifies the known flow features of the Colorado River at Lees Ferry. While a prerequisite for application in other cases, confirmation of what is already known does not justify applying the wavelet technique when existing methods provide the same information. We now provide four themes, using the Colorado River and additional information from the Snake River, to illustrate how the wavelet transformation can provide information that is not generated by traditional hydrologic analysis.

First, scale-averaged wavelet power can be used to extract wavelet power for a specific range of scales. Using the GWS, the statistical significance of this scale-averaged wavelet power may be calculated for all time periods. In the WPS calculated from both daily mean (Plate 1) and continuous (Plate 2) discharge, strong wavelet power existed at

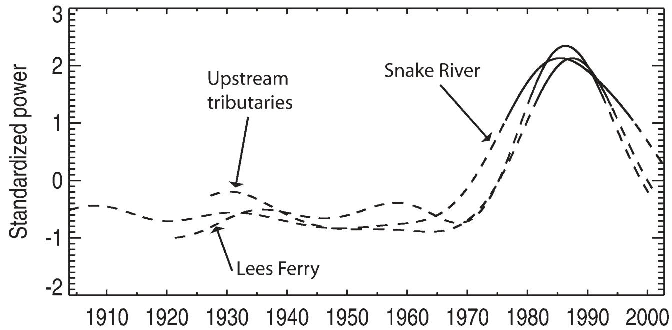

Figure 6. Scale-averaged wavelet power for the 9–12 year scale for three records: Lees Ferry, the combined major upstream Colorado River tributaries (see Figure 1), and the Snake River near Moran, Wyoming. Solid portion of each line shows the 95% confidence level based on the global wavelet spectrum. See Appendix for methodological details

roughly a ten-year scale from about 1970 to the end of the record. Much of this feature exists below the COI, but this would tend to artificially reduce, not increase, the WPS. This feature was also apparent in the WPS for the more natural upstream tributaries (not shown but available upon request), suggesting that the feature may not be entirely related to dam operation. To further investigate this decadal cycle, we used the WPS calculated from daily mean discharge at the Snake River near Moran, Wyoming. In this river system, which has been managed to reduce but not eliminate discharge amplitude at annual and six-month scales, the decadal feature was again present (not shown). We then extracted the 9 to 12 year scale-averaged wavelet power (see Appendix). Records from the upstream tributaries, Lees Ferry, and the Snake River all showed statistically significant wavelet power at the 9 to 12 year scale peaking in the mid-1980s (Figure 6). To our knowledge, this is the first analysis to illustrate the presence of this cycle in the western United States in systems with such dramatically different management strategies, suggesting that long-term streamflow cycles, probably related to climatic cycles, are not removed by dam operations.

Second, normalization of the full WPS (Plate 1) by the LWS of the pre-dam period can reveal discharge amplitude that is statistically larger than the expected undisturbed flow regime (Plate 3). This depiction is dramatically different than either the hydrograph (Figure 2a) or the WPS itself (Plate 1).

In the pre-dam period, the annual scale, when normalized by the LWS, did not experience a single episode of statistically significant power (Plate 3). This indicates that the annual scale, while very strong, was also highly consistent and, within 41 years of natural variability, was never much larger than expected. Brief significant amplitude modulation occurred at about a two-year scale in the early 1930s and 1960s but was otherwise absent. Frequent significant modulation existed throughout the pre-dam period at scales shorter than 200 days, indicative of variability in short-term events associated with weather variation and snowmelt dynamics.

The post-dam period had two modes of difference from the pre-dam LWS: very long and very short (Plate 3). The very long 9- to 12-year cycle, which is clearly unlike anything experienced in the pre-dam period, we have posited above is probably related to regional climate variability. The very short one- to seven-day cycles were not present in the WPS itself (Plate 1) but here, when normalized by the LWS, are apparent as highly distinct from the flow regime of the pre-dam period. This is a crucial distinction: the WPS (Plate 1) shows cycles that are statistically significant in comparison to a random noise process; the normalization by the LWS shows cycles that are statistically larger than a reference period, here the pre-dam record.

Third, normalization of the WPS by different LWS (e.g. pre- or post-dam) can reveal dramatically different features of the flow regime. Normalization of the post-dam continuous discharge WPS illustrates discharge patterns that are statistically different from both post-dam (Plate 4a) and pre-dam (Plate 4c) records. When normalized by the post-dam LWS (Plate 4b), large flow events at scales longer than 100 days occurred in association with the 1965 scouring floods and the mid-1980s large snowpack (Plate 4a). High discharge amplitude was pervasive from 0.1 to 10 days from 1970 to 1980, frequent from 1980 to 1991, and thereafter nearly absent, except for the 1996 experimental flood. When normalized by the pre-dam LWS (Plate 4d) the magnitude of the novel discharge patterns created by dam operation are much more obvious (Plate 4c). The one-day cycle was almost always much larger than the pre-dam LWS, often by more than a factor of 200. Even when not significant in the WPS (Plate 2), as

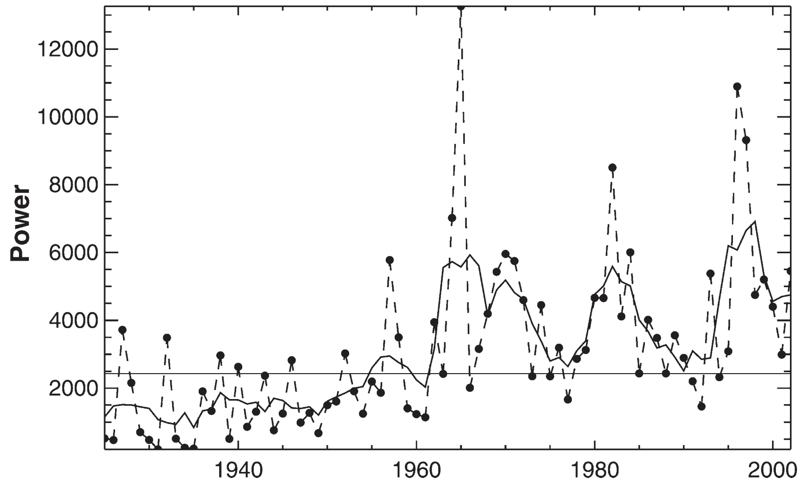

Figure 7. Snake River at Hell's Canyon 1 to 14 day scale-averaged wavelet power. Each symbol is the annual average. Solid curve is a five-year running average. Horizontal line shows the 95% significance level

during the post-1991 period, the daily cycle was still at least ten times larger than pre-dam daily amplitude. Thus, even in periods managed to minimize variation, the post-dam diurnal cycle was, in reality, vastly larger than the undisturbed diurnal cycle. Only in the brief 1997 spill avoidance periods was the daily cycle truly reduced. The 12-hour cycle is also readily apparent (Plate 4c) when normalized by the pre-dam LWS. By comparison, the weekly cycle is only moderately different than pre-dam patterns.

Fourth, wavelets can be used as an efficient data mining technique to identify streamflow variability. Hell's Canyon Dam on the Snake River was completed in 1968 and is operated so as to moderate the magnitude of the annual flood to a much lesser degree than Glen Canyon Dam. The WPS (not shown) indicates that the annual cycle was, if anything, stronger in the post-dam period. Although there are no generally known variations in dam operational rules, as there are for Glen Canyon Dam, normalizing by the pre-dam LWS suggested modulation in the WPS from about 1 to 14 days. We extracted this scale-averaged wavelet power and found that long-term cycles existed in the 1 to 14 day scale beginning with the mid-1950s construction of the upstream Brownlee dam (Figure 7). The five-year running mean shows an approximately decadal cycle. It is unlikely that modulation at the 1 to 14 day scale would be caused by long-term climate variability and is probably related to dam operations, details of which are unknown. This analysis shows how, for a dam system in which the timing of operational changes is not known, wavelet analysis can be used as an effective data mining tool.

Are there any practical ecological or management applications of this technique?
Yes.

The periodicity in flood and base flow leads to distinctive morphologic patterns on river beds and banks. Cyclic patterns create the opportunity for establishment of riparian forests. The life history strategies of many fish species depend on cues of the hydrologic regime. In this sense, wavelet analysis provides the opportunity to evaluate changes in these cycles and to encourage research that examines the linkages between these cycles and the characteristics of the physical and ecological attributes of the river systems in question. In the realm of dam re-operation, wavelets could be used to assess the success or failure of attempts to reconstruct certain features of pre-dam discharge characteristics at virtually any time scale. For example, if six-month cycles were required for a particular ecological restoration goal, wavelets would be an ideal assessment tool to evaluate how well dam operations created the feature.

For such assessments, we advocate normalizing the WPS by the LWS of a targeted reference period (e.g. Plates 3 and 4). If a certain range of years can be identified as favourable or unfavourable for a particular management goal, a LWS for this period should be constructed and used to normalize the WPS for identification of historical or current flow regimes with significantly higher amplitude. Although we have shown here a depiction of flow events that are statistically different (larger) from the expected value, a depiction of flow regimes similar to the reference LWS could be constructed easily. This approach is perhaps the best example of a practical implementation of wavelet analysis in a way that is both relevant to ecological research and difficult to accomplish with traditional hydrologic analysis.

Specific ecological and geomorphic systems exist in the Grand Canyon region in which this approach would be a useful component of an ecological analysis or experiment. Brown *et al.* (1998) found that the diurnal cycles caused by dam operation had created a quasi-tidal environment similar to that found in coastal regions and that consequently, rainbow trout became stranded in isolated pools, leading to excellent foraging habitat for bald eagles. Ironically, the study ended in March 1991 (Brown, 1993), thus narrowly missing an opportunity to directly test the impact of reduced diurnal flows. In marsh ecosystems, diurnal flow cycles are related to the establishment success of specific species in specific soil types and to the ability of riparian soils to retain plant litter (Stevens *et al.*, 1995). Drying cycles also affect populations of *Cladophora* species (Shaver *et al.*, 1997), which in turn can impact food web constituents. The stability of sandbars is related to diurnal wetting cycles (Budhu and Gobin, 1994, 1995) and could be studied in the context of the amplitude modulation shown in Plate 4c. Lastly, the final Environmental Impact Statement for the operation of Glen Canyon Dam (United States Department of Interior and Bureau of Reclamation, 1995) provides a description of the response of many ecological systems to various flow regimes.

CONCLUSIONS

The wavelet analysis reported here reveals the timing and structure of many management practices and changes in river hydrology, and is consistent with the known history of the effects of Glen Canyon Dam on the Colorado River. Indeed, we chose to assess this hydrologic record because Glen Canyon Dam operations are so well understood. We therefore confirm that wavelet analysis can be used to characterize dam operations and to detect different periods of dam management rules. We are confident that this technique could be used to assess hydrologic management in less well-understood rivers, as we have shown for the Snake River, or for dams operated for other purposes, such as water removal for irrigation, where the rules of dam operation are often unavailable. For wavelet applications in river systems, we present the following three concluding themes.

First, conclusions about dam operations are dependent on the time resolution of input data, a crucial point because continuous data are rarely available and many investigators analyse daily mean data instead, arguing that the weekly range of daily mean values is a sufficient surrogate for the hourly changes that typically occur at hydroelectric dams. At Glen Canyon Dam, statistically significant cycles were detected up to about 10 days in WPS calculated from daily mean discharge and up to 10 hours in WPS calculated from continuous data. Thus, assessment of dam operations without recourse to continuous data is likely to be flawed or incomplete.

Second, wavelet analysis can detect the hydrologic implications of management practices at all times and all scales without any *a priori* determination of the time scales of analysis. Thus all cyclic patterns of natural and regulated flow, and their differences, can be detected. WPS, for example, showed that the transition from traditional hydropower to environmentally driven experimental management of the dam in August 1991 reduced the strength of the one- and seven-day cycles and that the 1996 controlled flood had little effect in recreating the predam hydrologic cycle. Most significantly, no information on dam management is required to detect alterations to flows at any (possibly unpredictable) time scale.

Third, normalization of the WPS by LWS is a powerful data mining technique and is likely to provide the most useful depiction of modulations in discharge amplitude and scale. We normalized the post-dam continuous discharge WPS by its own LWS and by the pre-dam LWS and produced extremely different representations: we strongly advocate this type of normalization by relevant reference LWS.

In summary, when dam operational history is not well known, the time scales of relevant processes are unknown, or when the strength of discharge cycles relevant to one or more reference time periods is desired, wavelet analysis is an optimal tool with which to create a detailed and highly integrative depiction of river hydrology.

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APPENDIX

The wavelet technique is based on signal frequency variation and is highly visual yet mathematically based and statistically testable. Wavelet analysis is similar to Fourier analysis but instead of sine and cosine functions, employs one of a series of different wavelet functions which may consist of step functions (Haar wavelet), derivative of Gaussian functions, or a multi-peak Morlet wavelet composed of a sine wave superimposed on a Gaussian curve. Each function is composed of a scaling and detail wavelet. As Csillag and Kabos (2002) stated in their review of wavelets, the scaling wavelet is conceptually similar to an optical zoom feature while the detail wavelet represents optical focus.

Wavelets can be broadly considered in two categories: continuous and discrete. In the continuous wavelet transformation (CWT), the wavelet is translated throughout the input signal, often a vector containing time series data. At each point in the signal (translation), the wavelet is dilated between the finest and coarsest scales. For streamflow data in the United States, finest scales could range from 15 minutes to 24 hours while coarsest scales could extend to multiple decades. Due to the overlap at each translation point, the CWT is non-orthogonal. In the discrete wavelet transformation (often conducted with wavelet functions such as the Haar or Daubechies), the wavelet is dilated at dyadic resolutions, minimizing signal overlap at neighbouring scales and yielding a generally orthogonal wavelet. The CWT is well suited to analyse vector data while the discrete wavelet transformation is widely employed in two-dimensional image processing and image compression (White *et al.*, 2003).

The wavelet transformation W_n is the convolution of a vector x (with time dimension n) with a wavelet function Ψ

$$W_n(s) = \sum_{n'=0}^{N-1} x_{n'} \overline{\Psi\left[\frac{(n'-n)\delta t}{s}\right]} \tag{1}$$

where s is the scale, or dilation, $n' - n$ shows the number of points from time series origin (translation), δt is the time interval, N is the number of points, and the overbar designates the complex conjugate. Scale is the width of the wavelet: a larger scale means that more of the time series is included in the calculation and that finer details are ignored. Scale is approximately equal to Fourier period (inverse of frequency). Translation of the wavelet is accomplished by calculating the convolution from $n' = 0 \ldots N-1$. In other words, a wavelet of varying width (scale) is moved, or translated, through the entire time series. The wavelet transformation is therefore localized in both time (through the translation) and frequency (through the range of scales). Wavelets are advantageous in that they simultaneously localize frequency and time, allowing for the detection of variations in the amplitude and timing of periodic signals present in the time series.

For the current analysis, we used the complex Morlet wavelet function $\Psi_0(\eta)$, which is commonly used for signals with strong wave-like features (such as streamflow data):

$$\psi_0(\eta) = \pi^{-1/4} e^{i\omega_0\eta} e^{-\eta^2/2} \tag{2}$$

where ω_0 is the non-dimensional wave number (six to satisfy wavelet admissibility criterion of zero mean) and η is a time parameter (non-dimensional, also could represent other metrics such as distance).

The convolution shown in Equation 1 can be accomplished at all N based on a discrete Fourier transform

$$\hat{x}_k = \frac{1}{N} \sum_{n=0}^{N-1} x_n e^{-2\pi ikn/N} \tag{3}$$

where k is the frequency index. The wavelet transformation is then calculated as

$$W_n(s) = \sum_{k=0}^{N-1} \hat{x}_k \overline{\hat{\Psi}(s\omega_k) e^{i\omega_k n\delta t}} \tag{4}$$

where ω is the angular frequency and $\hat{\Psi}(s\omega)$ is the Fourier transform of $\Psi(t/s)$ in the continuous limit. In order to make wavelets intercomparable, the wavelet has been normalized

$$\hat{\Psi}(s\omega_k) = \sqrt{\left(\frac{2\pi s}{\delta t}\right)} \hat{\Psi}_0(s\omega_k) \tag{5}$$

such that

$$N = \sum_{k=0}^{N-1} |\hat{\Psi}(s\omega_k)|^2 \tag{6}$$

A consequence of division by δt in the normalization process is that wavelet power spectra calculated from time series of different time resolutions must be multiplied by $1/\delta t$. For example, the wavelet power spectrum calculated from hourly discharge must be multiplied by 1/24 in order to directly compare with the wavelet power spectrum from daily discharge.

The CWT can then be calculated using a fast Fourier transform (FFT) in any of several commonly available software packages such as Matlab or the Interactive Data Language. The wavelet power spectrum (WPS), as for the Fourier power spectrum, is defined as $|W_n(s)|^2$.

In addition to viewing the entire wavelet power spectrum, wavelets can be averaged in time and space. To average in time, i.e. to find the average wavelet power at all scales for $n = 0 \ldots N$ one simply calculates the global wavelet spectrum (GWS):

$$\overline{W}^2(s) = \frac{1}{N}\sum_{n=0}^{N-1} |W_n(s)|^2 \tag{7}$$

Or, if a local wavelet spectrum (LWS) is desired for a particular range of times from $n = n_1$ to $n = n_2$ then, for the number of times averaged, n_a,

$$\overline{W}^2(s) = \frac{1}{n_a}\sum_{n=n_1}^{n_2} |W_n(s)|^2 \tag{8}$$

The scale-averaged wavelet power is a scale-weighted slice from the WPS for a certain range of scales of interest $(j_1 \ldots j_2)$:

$$\overline{W}_n^2 = \frac{\delta j \delta t}{C_\delta}\sum_{j=j_1}^{j_2} \frac{|W_n(s_j)|^2}{s_j} \tag{9}$$

where δj is the scale sampling interval, C_δ is an analytically determined wavelet reconstruction factor (0.776 for the Morlet with wavenumber of 6).

Torrence and Compo (1998) showed that both the Fourier power spectrum and wavelet power spectrum follow a chi-squared distribution with two degrees of freedom. Assuming a random process, such as red noise, the theoretical background spectrum of a time series can then be calculated. Then, for any significance level from the chi-squared distribution, one can then construct confidence level contours to superimpose on the wavelet power spectrum. This approach shows wavelet power that is significantly different from random noise. We selected the 95% confidence interval for wavelet power as our criteria for significance.

Alternatively, one could use the global wavelet spectrum, $\overline{W}^2(s)$, as the theoretical background spectrum and construct a similar confidence interval. The previous confidence interval would then show wavelet power significantly different from a random process while use of $\overline{W}^2(s)$ would show the scale and time of wavelet power statistically different from the 'normal' wavelet power, defined either as a local or global wavelet spectrum. This latter technique provides an easy way of detecting the location of significant frequency and amplitude modulation.

For efficiency in the FFT, the time series is padded with zeroes to bring N up to the nearest power of two, but data at the beginning and end of the time series will then include increasing numbers of zeroes and thus lower wavelet power, especially at wide s. This introduces discontinuities. The 'cone of influence' based on an empirically determined Fourier wavelength for the Morlet wavelet with a wavenumber ω_0 of 6 is:

$$\left[\frac{\left(\frac{4\pi}{\omega_0+\sqrt{2+\omega_0^2}}\right)}{\sqrt{2}}\right]\delta t[0, 1, \ldots, ((N+1)/2), (N/2), (N/2)-1, \ldots, 0] \tag{10}$$

To illustrate the conceptual link between discharge cycles and the WPS, we generated four hypothetical 10-year discharge records with hourly resolution (Plate 5): (1) purely diurnal period with low amplitude; (2) purely diurnal period with high amplitude; (3) purely annual period with low amplitude; and (4) purely annual period with high amplitude. While the true benefits of wavelet analysis, frequency and time localization are not shown, the reader can gain an intuitive understanding of the connection between discharge period and amplitude and the WPS.

