# Slope Instability from Ground-Water Seepage

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**Abstract:** Previous researchers have treated the magnitude and direction of the seepage vector as independent variables and found that seepage, parallel to the surface, does not result in the minimum stable seepage slope. In the present paper, the magnitude and direction of the seepage vector is shown to be uniquely related at the seepage face, and seepage parallel to the slope results is the minimum stable seepage slope. Slope failures under seepage are found to be progressive, and the stable slope angle depends on the direction of the seepage vector. The effect of seepage direction on static liquefaction is investigated, and the seepage directions that initiate static liquefaction depend on the slope angle and soil-unit weight. Analytical predictions of the minimum stable seepage slope compare favorably with experimental results.

## INTRODUCTION

Ground-water seepage has often resulted in catastrophic slope failures. River banks, canal and reservoir embankments, and hillside slopes exemplify situations where seepage erosion has been observed. In this study, a two-dimensional (2D) limiting equilibrium approach is used to analyze the minimum, stable seepage-slope angle for an infinite, cohesionless slope under a steady-state seepage regime. Iverson and Major (1986) found that minimum slope stability for cohesionless soils occurs when the seepage vector is horizontal. In the present paper, seepage parallel to the slope defines minimum stability. Slope failures in sand depend on the direction of the seepage vector.

## ANALYSIS OF SEEPAGE EROSION

The maximum stable slope angle for dry sand, under no external load, is its angle of internal friction ($\phi$). If seepage is permitted through the sand mass, it will collapse to a smaller stable slope (stable seepage slope). Consider a 2D soil element within a homogeneous infinite slope with stress-free boundaries and slope angle $\alpha$ (Fig. 1). The seepage vector of magnitude $i$ (hydraulic gradient) is assumed to be inclined at angle $\lambda$, measured clockwise from the inward normal to the slope (Fig. 1). Emergent seepage will impose a force per unit width, $i\gamma_w A$, on the element where $\gamma_w$ = unit weight of water; $A = bz$ = area of the element; $b$ = length of the element parallel to the slope; and $z$ = height of the element (Fig. 1). From the equilibrium of an elemental soil volume (Fig. 1), using Coulomb failure criterion with the failure plane parallel to the slope, the factor of safety against failure is

$$F = \frac{[(\gamma'/\gamma_w)\cos\alpha - i\cos\lambda]\tan\phi}{(\gamma'/\gamma_w)\sin\alpha + i\sin\lambda} \qquad (1)$$

Now, consider a flow net within a slope as shown in Fig. 2. On seepage face $AB$, $\lambda = 90°$ at $A$, and $\lambda = 0°$ at $B$. The valid range of $\lambda$ on seepage face $AB$ is, therefore, $90° \geq \lambda \geq 0$. Harr (1962) showed that the velocity at point $C$ on seepage surface $AB$ (Fig. 2) is

$$v = k\sin\alpha/\sin\lambda \qquad (2)$$

and from Darcy's law we obtain

$$i = \sin\alpha/\sin\lambda \qquad (3)$$

By substituting (3) into (1), we obtain

$$F = \frac{[(\gamma'/\gamma_w)\cos\alpha - \sin\alpha\cot\lambda]\tan\phi}{\sin\alpha(\gamma'/\gamma_w + 1)} \qquad (4)$$

At the seepage face, flow is parallel to the slope, and the hydraulic gradient is greater than at any other point within the flow domain. Thus, failures caused by seepage will be initiated along or near the seepage face. The factor of safety of a slope must be at least one (limiting equilibrium). From (4), for positive values of $F$, we obtain

$$(\gamma'/\gamma_w)\cos\alpha - \sin\alpha\cot\lambda \geq 0 \qquad (5)$$

By simplification, (5) becomes

$$\tan\lambda \geq (\gamma_w/\gamma')\tan\alpha \qquad (6)$$

The valid range for $\lambda$, for a Coulomb failure, is then $\tan^{-1}(\gamma_w/\gamma' \tan\alpha) \leq \lambda \leq 90°$ (Fig. 1). At limiting equilibrium $F = 1$, (1) reduces to

$$\tan\phi = \frac{(\gamma'/\gamma_w)\sin\alpha + i\sin\lambda}{(\gamma'/\gamma_w)\cos\alpha - i\cos\lambda} \qquad (7)$$

By subsitituting (3) into (7), we obtain

$$\tan\phi = \frac{\sin\alpha(\gamma'/\gamma_w + 1)}{(\gamma'/\gamma_w)\cos\alpha + \sin\alpha\cot\lambda} \qquad (8)$$

For seepage parallel to slope $\lambda = 90°$, (8) further reduces to Taylor's (1948) equation

$$\alpha = \tan^{-1}[(\gamma'/\gamma_{sat})\tan\phi] \qquad (9)$$

For many soils, a reasonable approximation is $\gamma'/\gamma_w = 1$, which by subsitution in (8) results in

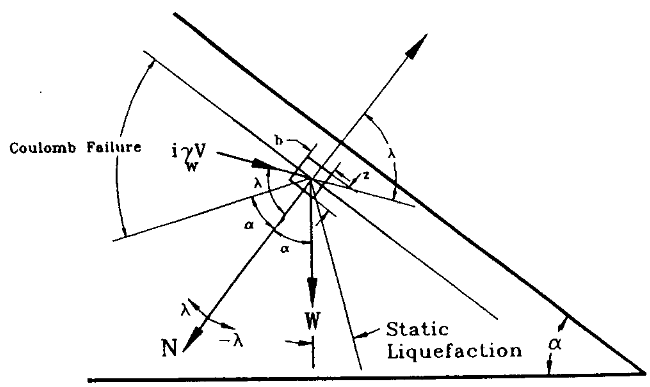


**FIG. 1. Forces on Elemental Volume of Soil and Range of λ for Coulomb Failure and Static Liquefaction**

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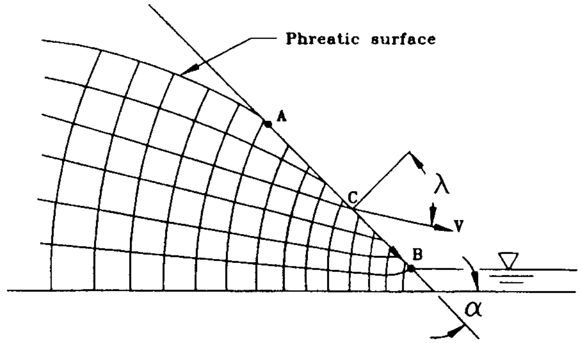


**FIG. 2. Flow Field in Homogeneous Isotropic Soil Showing Exit Velocity and Its Components along Seepage Face AB**

$$\tan \alpha = \tan \phi/(2 - \tan \phi \cot \lambda) \tag{10}$$

A plot of (10) for $\phi = 30°$ is shown in Fig. 3 for the valid range of $\lambda$ for emergent seepage at the seepage face, i.e., from a maximum value of 90° to $\tan^{-1}(\gamma_w/\gamma' \tan \alpha) = \tan^{-1}(1 \tan 30°) = 30°$; the corresponding values of $i$ are also shown. Only a single curve is generated for each equation; a family of curves is not, as described by Iverson and Major (1986). For example, if $\alpha = 30°$ and seepage is horizontal, then $\lambda = (90 - \alpha) = 60°$, and $i = 0.577$ in (3). Only one value of $i$ is consistent with this direction of seepage. The value of $i$, corresponding to the appropriate value of $\lambda$, is shown in Fig. 3 for emergent seepage at the seepage face. The hydraulic gradient at the seepage face, as expected from (3), increases from 1 (when $\lambda = \alpha$) to its minimum value of $\sin \alpha$ (when $\lambda = 90°$).

The minimum stable seepage slope occurs when $\lambda = 90°$, i.e., seepage is parallel to the slope. Iverson and Major (1986) proposed that minimum slope stability will universally occur when $\lambda = 90° - \phi$. Later, Iverson (1992) stated that $\lambda = 90° - \phi$ "does not rigorously apply to subaerial infinite slopes." According to Iverson and Major (1986), a slope originally at its angle of friction ($\alpha = \phi$) will achieve minimum slope stability when seepage is horizontal.

Iverson and Major [(1986) Figs. 3 and 4], show that $\alpha$ can exceed $\phi$. They assumed that the slope angle in a cohesionless soil can lie within the range $0 < \alpha < 90°$. The upper limit of this range is invalid for a Coulomb failure. In the absence of seepage forces, the maximum slope angle for a cohesionless soil is its angle of friction, $\phi$. The range of the stable slope angle is then $0 < \alpha < \phi$. The slope angle, $\alpha$, can exceed $\phi$, if seepage is directed into the slope.

In Fig. 3, the slope, originally at its angle of friction ($\alpha = \phi = 30°$), will fail once $\lambda$ exceeds 30° at the seepage face. For example, if the predominant seepage direction is $\lambda = 60°$ (horizontal seepage), for which $i = 0.577$, the slope will collapse from $\alpha = 30°$ to $\alpha = 19°$. The stable seepage slope decreases with increasing $\lambda$, reaching its minimum value for a Coulomb failure, when the predominant seepage direction is parallel to the slope ($\lambda = 90°$). Slope failures influenced by emergent seepage are therefore progressive, and the minimum stable seepage slope is reached when $\lambda = 90°$.

## STATIC LIQUEFACTION

For static liquefaction, the vertical component of the seepage force must be equal to or greater than the weight of the soil. Resolving forces (seepage and soil weight per unit width) vertically (Fig. 1), we obtain

$$W = i\gamma_w A \sin(\lambda + \alpha - 90) = i\gamma_w A \cos(\lambda + \alpha) \tag{11}$$

By substituting and rearranging $W = \gamma' A$ and $i = \sin \alpha/\sin \lambda$ in (11), the results are

$$(\cos \alpha + \cos \lambda)\frac{\sin \alpha}{\sin \lambda} = \frac{\gamma'}{\gamma_w} = \frac{G - 1}{1 + e} = (1 - n)(G - 1) \tag{12}$$

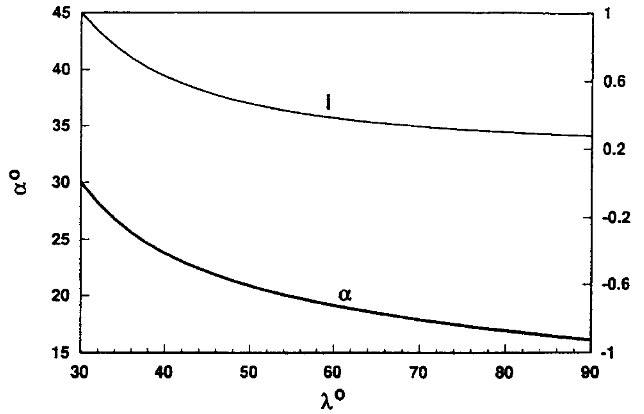


**FIG. 3. Slope Angles for Different Seepage Directions and Corresponding Hydraulic Gradient for $\phi = 30°$**

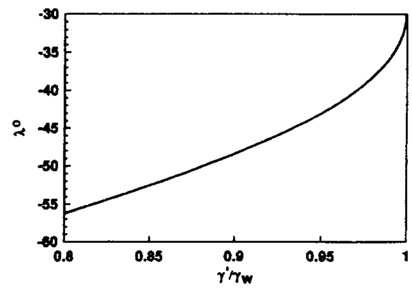


**FIG. 4. Variation of Seepage Direction for Static Liquefaction as Function of $\gamma'/\gamma_w$**

where $G$ = specific gravity of the soil; $e$ = void ratio; and $n$ = porosity.

For most soils, $\gamma'/\gamma_w = 1$ and the solution of (12) is $\lambda = -\alpha$. Therefore, static liquefaction for most soils ($\gamma'/\gamma_w = 1$) will take place when seepage is vertically upward, corresponding to a hydraulic gradient of $-1$. For soils with effective unit weight ratio $\gamma'/\gamma_w$ other than 1, static liquefaction will occur at seepage directions determined from (12). A plot of (12) is shown in Fig. 4 to define the directions of seepage that will induce liquefaction for variations in $\gamma'/\gamma_w$ normally obtained in practice. The practical range of $\lambda$ that will cause liquefaction is shown in Fig. 1.

## COMPARISON OF ANALYSIS WITH LABORATORY EXPERIMENTS

A sand mass was deposited in water over a 3 m length in a flume 5.5 m long, 0.61 m wide, and 0.76 m high. A slope at an angle $\alpha = 32°$ was constructed on one face of the sand mass in the flume. The properties of the sand, determined from the following laboratory tests in accordance with ASTM are:

- Grain size (ASTM D421-422)—average grain size, $D_{50}$ = 0.73 mm, coefficient of uniformity $C_u = 3.9$
- Unit weight (ASTM D1556-82)—$\gamma_{sat} = 19$ kN/m$^3$
- Constant head-permeability test (ASTM D2424-68)—coefficient of permeability, $k$, = $5.0 \times 10^{-3}$ cm/s
- Shear box test (ASTM D3080-90)—$\phi = 32°$

One of the longitudinal sides of the flume was constructed from glass enabling observations and measurements of slope changes caused by ground-water seepage.

The external water level (water level in front of the slope) was raised to the top of the slope and kept there until equilibrium was achieved with the ground-water level on the slope. As the external water level was lowered at a rate of 0.1 m/min (maximum withdrawal rate permitted by the outflow valve), cracks appeared on the slope. When the external water

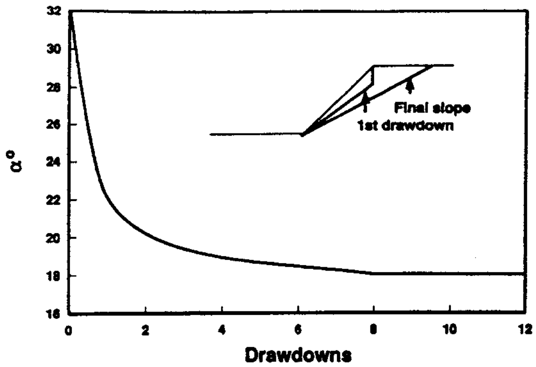


FIG. 5. Observed Stable Seepage-Slope Angles as Function of Number of Drawdowns

level reached an elevation 7.5 cm from the base of the slope (low water level), the slope failed. A vertical face, 27 cm in depth from the top of the slope, was followed by a slope of 22° after the failure (Fig. 5). The preceding procedure of raising and lowering the external water level was repeated several times. During the second rise of the external water level, the vertical face collapsed indicating that this face was formed by capillary action. Further slope failures were observed until the slope eventually stabilized at an angle of 18° (Fig. 5).

The results indicate that slope failures under seepage are progressive until a minimum stable seepage slope is achieved. From (9), the predicted minimum stable seepage slope for the sand mass is $\alpha_{min} = \tan^{-1}\{[(19 - 9.81)/19]\tan 32°\} = 17°$, which is in good agreement with the observed value of 18°. While scaling effects and boundary conditions are likely to influence the test results, these were judged to be negligible, since observations of failures on sandbars in the Grand Canyon were similar to the experimental results (Budhu and Gobin 1994). A more detailed series of experiments was conducted by Amanullah (1993), using various initial slopes, and rates of rise and drawdown of the external water level resulted in similar observations and conclusions to the preceding.

## CONCLUSIONS

The results of the analysis presented in this paper provide bounds on the seepage direction that provoke slope failures by Coulomb mechanism. Slope failures resulting from seepage forces are progressive, and the minimum stable seepage slope is reached when seepage is parallel to the slope. The hydraulic gradient and the seepage direction are shown to be uniquely related at the seepage face and are not independent variables, as was previously assumed. The seepage direction that initiates static liquefaction depends on the slope angle and the soil unit weight. For most soils, static liquefaction will occur when seepage is directed vertically upward.

## ACKNOWLEDGMENTS


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## APPENDIX I. REFERENCES


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## APPENDIX II. NOTATION

*The following symbols are used in this paper:*

$A$ = area of soil element;
$b$ = length of soil element parallel to slope;
$C_u$ = coefficient of uniformity;
$D_{50}$ = average grain size;
$e$ = void ratio;
$F$ = factor of safety against Coulomb failure;
$G$ = specific gravity;
$i$ = magnitude of seepage vector;
$k$ = coefficient of permeability;
$N$ = normal force on elemental soil volume;
$n$ = soil porosity;
$W$ = weight;
$z$ = height of element;
$\alpha$ = slope angle;
$\gamma'$ = effective (submerged) unit weight of the soil;
$\gamma_{sat}$ = saturated unit weight of the soil;
$\gamma_t$ = total unit weight of the soil;
$\gamma_w$ = unit weight of water;
$\lambda$ = direction of the seepage vector; and
$\phi$ = angle of internal friction.