PATTERNS OF SCOUR AND FILL IN POOL-RAPID RIVERS

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ABSTRACT

Because the change in head on a critical control (rapid or weir) is less than the change in equilibrium depth in the pool upstream, a pool will eventually scour with an increase in flow and fill with a decrease in flow. However, because the supply of sediment to a pool is dependent on the conditions at the outlet of the pool just upstream, initially there may be either scour or fill in the downstream pool with an increase in flow. Indeed, a pool following several other pools may behave in a seemingly erratic manner because its sediment supply is affected by what happens in each of the upstream pools.

INTRODUCTION

The pool-rapid river consists of short and medium length reaches at subcritical flow separated by rapids where the flow rate is supercritical. The Colorado River through the Grand Canyon is a prime example of this kind of river, and there are many others.

Since each reach has a downstream control, a natural first impression is that each reach should be hydraulically independent except for the attenuation of the hydrograph by the reservoir effect—assuming, of course, that the controls are not drowned out at high water. However, because sediment is transported in a real river, the reaches are not independent; the behavior of any reach is dependent on what happens in (and, therefore, on the characteristics of) all upstream reaches.

The characteristics of the imaginary pool-rapid river studied in this investigation were completely arbitrary—but not unlike the Colorado River. All reaches were 16 kilometers (10 miles) long. The cross sections were assumed rectangular with widths of 120 or 180 meters (400 or 600 feet). The banks were considered to be inerodible rock walls, as is true in much of the Grand Canyon. The erodibility of the bank in general is a factor determining the width of a stream, and widths can change during extreme floods if the banks are erodible. These effects were put aside in this investigation. A slope of 0.00005 and an n value of 0.03 were used as these gave velocities and depths which seemed reasonable.

The controls were approximated by either a parabolic or triangular weir giving different, but reasonable, head-discharge relationships.

$$Q = 55.8\ h^2 \tag{1}$$

$$Q = 28.4\ h^{2.5} \tag{2}$$

where Q, the discharge, is in cubic meters per second, and h, the head, is in meters.

SOME CHARACTERISTICS OF SEDIMENT-TRANSPORTING STREAMS

Alluvial streams transport sediment as well as water. Figure 1 illustrates some of the resulting stream behavior for a bed material typical of the Colorado River, several widths and slopes and an n value of 0.03. The Laursen (1958) relationship was used in the preparation of this figure but with the hydraulic radius R used to evaluate the shear velocity term and in the Manning equation. The width was found to be surprisingly ineffective as a determining factor in the load-discharge relationship. Indeed, this implies that the slope and sediment composition (and probably the Manning n) together with the "dominant" discharge might be sufficient to estimate the watershed sediment yield of bed material. An estimate of the change in bed material due to "wash load" during storms would be needed to guess at the total sediment yield.

Figure 1 also indicates that the slope in a long contraction is not much different than the slope of the uncontracted stream. Using the DuBoys and Manning equations and the approximations that hydraulic radius equals depth ($R = y$) and critical tractive force is much smaller than the total shear ($\tau_c \ll \tau_0$), Straub (1940) obtained an expression for the depth ratio as a power function of the width ratio of the uncontracted and contracted reaches.

$$y_2/y_1 = (B_1/B_2)^{9/14} \quad (3)$$

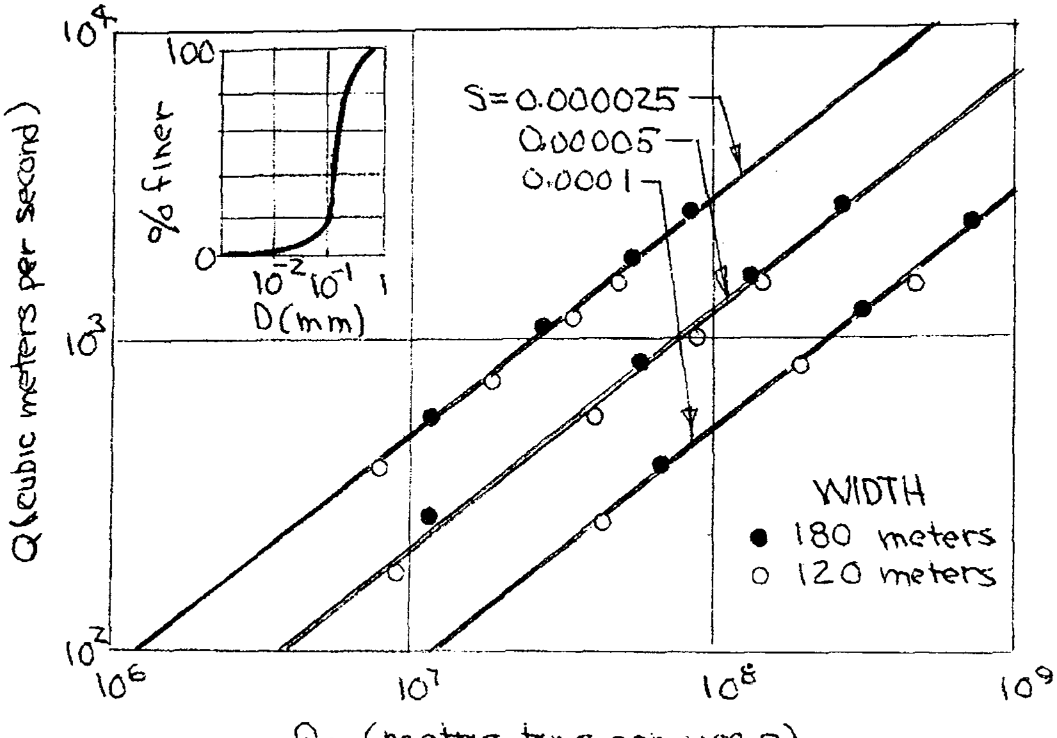

FIGURE 1. LOAD-DISCHARGE RELATIONSHIPS

This analysis can be extended to find a similar expression for the slope ratio, assuming n is the same in the two reaches,

$$S_1/S_2 = (B_1/B_2)^{1/7} \tag{4}$$

Letting $R = Ky$ and $(\tau_0'/\tau_c)-1 = C(\tau_0'/\tau_c)$ and approximating the shear velocity/fall velocity function in the Laursen relationship by power functions with the exponents 1/4, 1, and 9/4 when that velocity ratio is <1/2, =1, and >2, similar expressions result: (Note that τ_0' is the particle shear, not the total shear τ_0).

$\sqrt{gRS}/w < 1/2$

$$S_1/S_2 = (C_2/C_1)^{0.88}(K_2/K_1)^{1.30}(n_1/n_2)^{1.78}(B_1/B_2)^{-0.02} \tag{5}$$

$$y_2/y_1 = (C_2/C_1)^{0.26}(K_2/K_1)^{-0.01}(n_1/n_2)^{-0.07}(B_1/B_2)^{0.59} \tag{6}$$

$\sqrt{gRS}/w = 1$

$$S_1/S_2 = (C_2/C_1)^{0.71}(K_2/K_1)^{1.21}(n_1/n_2)^{1.29}(B_1/B_2)^{0.14} \tag{7}$$

$$y_2/y_1 = (C_2/C_1)^{0.21}(K_2/K_1)^{-0.04}(n_1/n_2)^{-0.21}(B_1/B_2)^{0.64} \tag{8}$$

$\sqrt{gRS}/w > 2$

$$S_1/S_2 = (C_2/C_1)^{0.54}(K_2/K_1)^{1.13}(n_1/n_2)^{0.78}(B_1/B_2)^{0.31} \tag{9}$$

$$y_2/y_1 = (C_2/C_1)^{0.16}(K_2/K_1)^{-0.06}(n_1/n_2)^{-0.37}(B_1/B_2)^{0.69} \tag{10}$$

K and C values are shown in Figure 2. Although the exponents of the C, K, and n ratios are in the order of 1, the ratios themselves should not be too different from 1. Moreover, $C_2>C_1$ while $K_2<K_1$ and probably $n_1>n_2$ (unless they are equal), so in general the slope in the contraction is slightly less than in the uncontracted (normal) stream. The difference in the slopes is small, and the difference in the difference with changing discharge should be even smaller. Slope changes are probably seldom observable except during the active period of scour and fill before equilibrium depths and velocities are attained.

Width changes have a considerably greater effect on depth; and not only is the depth greater in the contraction, but with an increase in discharge the contraction scours and the expansion beyond fills (and vice versa with a decrease in discharge). The exponents on the K, C, and n ratios are such that the effects of these ratios are generally negligible.

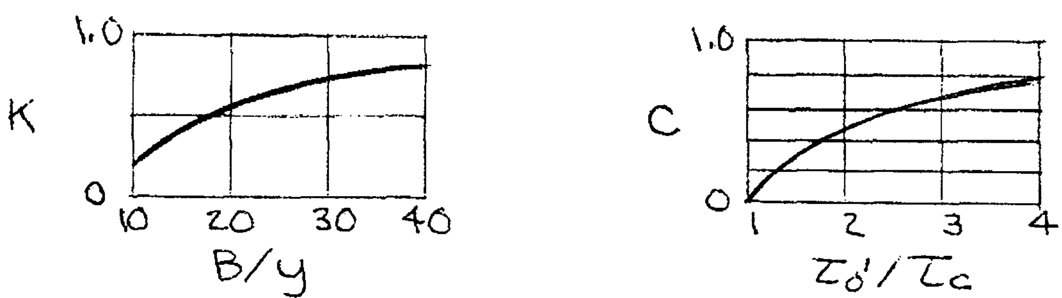

FIGURE 2. K & C VALUES

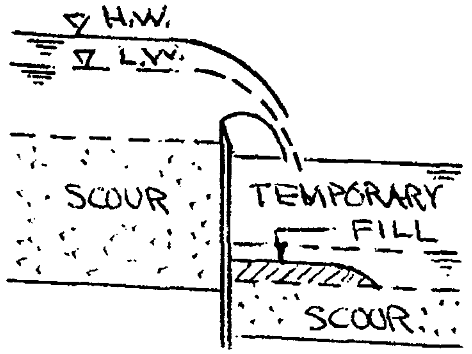

FIGURE 3, POOL-RAPID-POOL

The special characteristic of the pool-rapid reach is that the increase in head is not as great as the increase in equilibrium depth when there is an increase in discharge. As shown in Figure 3, the pool eventually scours out with an increase in flow (and fills with a decrease in flow). However, if the control or the width of the next pool downstream is different, the next pool may fill instead of scour, or scour to an interim equilibrium depth while the upstream pool is actively scouring and, therefore, delivering a non-equilibrium supply of sediment to the downstream pool. If the particular reach in question follows several pool-rapid reaches, the bed can fill and scour in what seems to be an inexplicable fashion without further change in the discharge. If the hydrograph goes up and down faster than the reaches can scour and fill to equilibrium, the pattern of scour and fill can be even stranger because residual holes and humps will tend to move through the system.

The time for a flood wave to pass through the system is approximately the quotient of the length of the system divided by the velocity of flow; minutes, hours, or days depending on the particular river. The time to reach equilibrium as a result of scouring or filling is approximately the quotient of the total volume of scour or fill divided by the difference between the supply of sediment from upstream and the capacity to transport sediment in the reach or reaches under examination; days, weeks, months, or even years depending on the particular situation.

In order to better understand the behavior of the pool-rapid river, several simpler cases were studied, each illustrating some facet of the more complex problem.

CASE I. A SET OF INERODIBLE LONG CONTRACTIONS AND EXPANSIONS

To test the degree of nonuniformity of flow after the flood wave has passed but before much of the scour and fill has taken place, the set of contractions and expansions shown in Figure 4 was examined as if the bed and bank were fixed in the equilibrium configuration associated with the previous low water flow. The reaches were assumed 16 km (10 miles) long except the first and last which were very long. With a low flow rate of 140 cubic meters per second (5000 cfs), a slope of 0.00005 in the expansions and an n value of 0.03, the depth in the 180 meter (600 ft) wide expansion reaches was 2.1 meters (6.8 ft) and in the contractions 2.5 meters (8.3 ft). At a flow of 700 m^3/ sec (25,000 cfs) the normal depth of flow in the expansions is 5.5 meters (18 ft) and in the contractions 7.1 meters (23.3 ft).

The normal depth of flow in the last long expansion would be the control for the system. The losses at the transitions between reaches and the changes in velocity head are small enough to be neglected. Therefore, the depth at the end of the upstream contraction would be 5.9 meters and would rise in a drop-down backwater curve to 6.4 meters at the beginning of the contraction—less than the normal depth. In the middle expansion the depth at the lower end

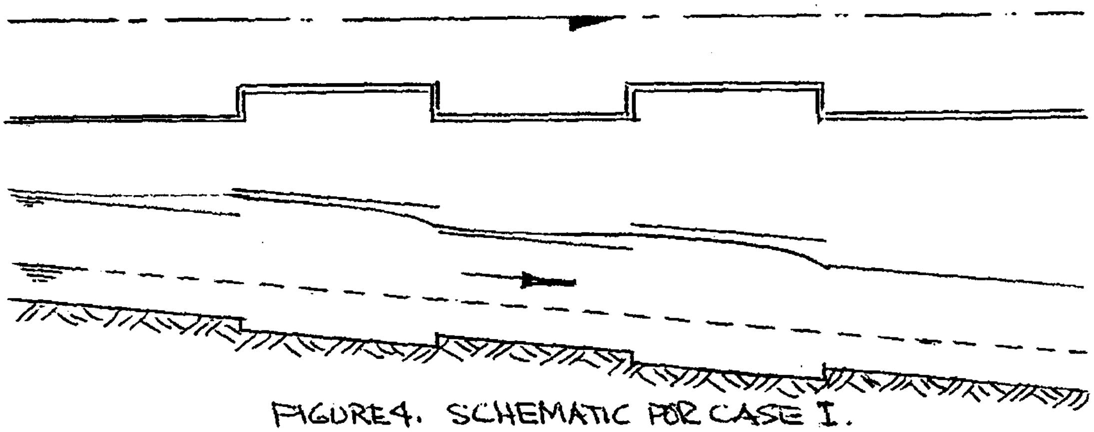

FIGURE 4. SCHEMATIC FOR CASE I.

would be 5.9 meters which would reduce to 5.8 meters at the beginning of the reach. In the next contraction upstream, the depths would be 6.2 at the tail end and 6.6 at the head end. At the tail end of the first expansion, the depth would be 6.2 meters and there would be a long backwater curve reducing the depth to the normal depth of 5.5 meters. Note that only in the downstream, control reach is the flow uniform. Otherwise, everywhere in the system there are differences in sediment-transporting capacity and a tendency to scour or fill.

CASE II. A SET OF ERODIBLE LONG CONTRACTIONS AND EXPANSIONS

If the bed of Case I is erodible and the stream is carrying a sediment from upstream, when the flow is increased there will be scour and fill until the depths and velocities are such that the same capacity to transport sediment exists everywhere. Because of the nonuniformity of the flow, there will be some reworking of the bed; however, this effect will disappear in the general scour in the contractions and fill in the expansions. The equilibrium depths would be 5.5 meters and 7.1 meters (23 ft) in the expansions and contractions. As shown in Figure 5, the material scoured out of the contractions would fill the next expansions. The fill in the last long expansion would gradually be carried on downstream, getting thinner and thinner so that it practically disappears. This would lower the control and all other expansions and

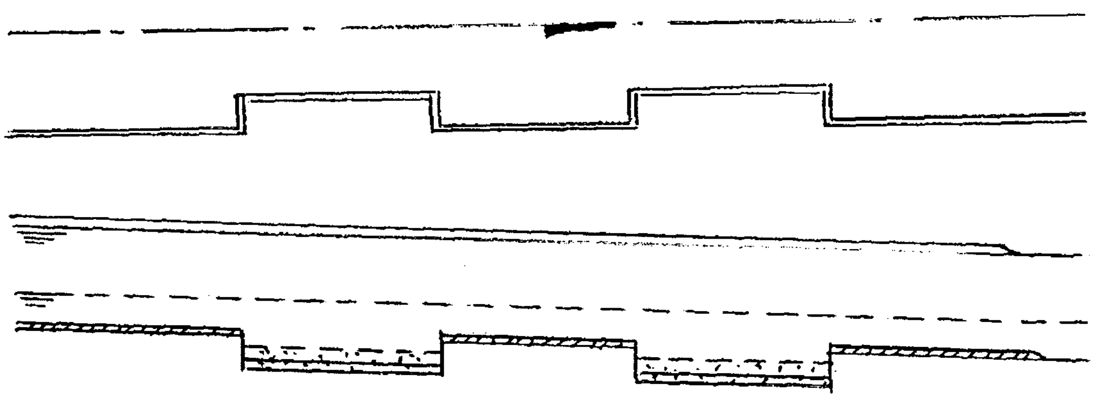

FIGURE 5. SCHEMATIC FOR CASE II.

contractions would gradually lower. At first there would be a scour of about 0.7 meters and a fill of about 0.5 meters. Eventually the scour would be 1.2 meters and there would be little or no fill.

If the bed material is typical of the Colorado River, the sediment supply from upstream at a discharge of 700 m^3/sec would be 50x10^6 metric tons per year. Because of the backwater, initially the supply rate to the first contraction would be only 27x10^6 metric tons per year. The initial sediment-transporting capacity at the head of the first contraction would be 86x10^6 metric tons per year. It would therefore take something more than 14 days for the contraction to scour out. Values of supply and capacity for the other reaches would be about the same and scour and fill would be occurring in all reaches simultaneously. For all the adjustments to occur, probably a month rather than two weeks would be needed.

CASE III. A SET OF POOLS WITH THE SAME WIDTHS BUT DIFFERENT CONTROLS

Consider first the simplest set of pools, all the same width (say 120 meters), the same length (16 km) except the first pool which is very long and all with the same control (parabolic weir). At a flow of 140 m^3/sec, the head on the control is 1.5 meters and the equilibrium depth is 2.5 meters, so the bed is 1.0 meters below the control crest. At a flow of 700 m^3/sec, the head on the weir is 3.4 meters and the equilibrium depth is 7.1 meters, so the bed ultimately will be 3.6 meters below the crest, thus the scour in each pool will be 2.6 meters (8.5 ft). Shortly after the flow has changed, the depth of flow at the tail end of each pool will be 4.4 meters while, because of backwater, the depth of flow at the head end of the pools will be 5.9 meters. Each pool will deliver more sediment than the next downstream pool can handle. Therefore there will be deposition at the head of each pool. Within each pool initially the capacity to transport sediment will be greater at the tail end than the supply coming from the head end. Therefore there will be scour at the tail end of each pool. The wedge shape of fill and of scour will result in a greater slope, uniform flow at a lesser depth and greater velocity, and a greater capacity to transport sediment than the ultimate equilibrium conditions. This will happen simultaneously in all pools and a temporary equilibrium will have been established. However, in the first long pool, there is no deposition and the sediment supply from upstream is that of the final, ultimate equilibrium condition. Scour will continue at the tail end of this long pool and the sediment supply to the second pool will decrease. The head end of the second pool will then scour also, and the scour hole will get deeper and longer until the entire pool has scoured to the equilibrium condition. Nothing will happen in the next downstream pool until the scour hole in the upstream pool reaches the control at the tail end; then the next downstream pool will begin to scour in the same way. Each downstream pool will be delayed until the pool just upstream has scoured out. Thus even for the simplest geometry, the essential character of pool-rapid river behavior is evident, and if the hydrograph changes before the entire system has attained equilibrium, the behavior will be even more complex.

Case III, as shown in Figure 6, is changed from this simple case in that the first and third pool controls are triangular weirs, whereas the second and fourth pools are controlled by parabolic weirs. For a flow of 1400 m^3/sec (50,000 cfs), the equilibrium depth for all pools is 11.0 meters; the head on

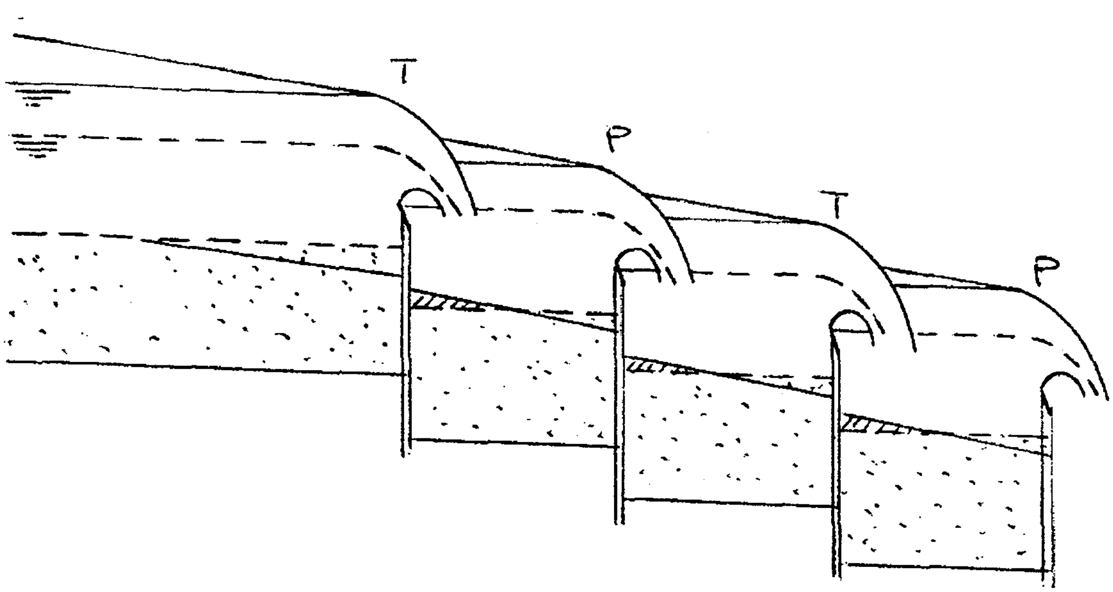

FIGURE 6. SCHEMATIC FOR CASE III.

the parabolic weirs is 4.8 meters, on the triangular weirs is 4.9 meters. The bed is 6.2 meters below the crest of the parabolic weir and 6.1 meters below the crest of the triangular weir. For a flow of 5600 m^3/sec, the equilibrium depth in all pools will be 27.1 meters; the head on the parabolic weir 9.1 meters and on the triangular weirs 9.7 meters. Ultimately the pools with the parabolic weirs will scour out 11.8 meters, and the pools with the triangular weirs will scour out 11.3 meters.

Immediately after the high flow is established, the depth at the tail end of the first long pool will be 15.8 meters and the depth at the head end of the next pool will be 18.3 meters. Therefore the head of the second pool will fill as the tail of the first pool scours. The initial depth at the tail of the second pool will be 15.3 meters; therefore the tail of the second pool scours. The heads of the third and fourth pools will also fill and the tails scour. Because the controls are different, the amount of scour and fill in each pool will not be quite the same and the time needed to attain this temporary equilibrium condition will vary a little. As in the simple case, the first pool continues to scour toward equilibrium and, as the supply to the second pool decreases, the head end begins to scour towards equilibrium. With only slight variations, the progress of the scour through the successive pools and from pool to pool will be the same as in the simple case. It is conceivable that the controls could be different enough so the depth at the head of one pool would be equal to or less than the depth at the tail end of the next upstream pool. In this circumstance, the head of the pool would not first fill but would hold steady for a while or immediately begin to scour. It is also conceivable that the controls could be so different that a downstream pool would exhibit very erratic behavior as the sediment supply originating in various upstream pools reached it.

CASE IV. A SET OF POOLS WITH DIFFERENT WIDTHS AND THE SAME CONTROLS

This case differs from the previous one in that the heads on the controls for the pools are the same, but the equilibrium depths are different because the widths of the pools are different. The situation is shown in Figure 7: in the 120 meter width pools, the equilibrium depth at 1400 m^3/sec is 16.0 meters and 27.1 meters at 5600 m^3/sec. In the 180 meter width pools, the equilibrium depth at 1400 m^3/sec is 8.5 meters and 19.8 meters at 5600 m^3/sec. The head on the controls at the low flow is 4.9 meters and at high flow is 9.7 meters. In the narrow pool, the bed is 6.1 meters below the crest at low flow and 17.4 meters at the high flow; therefore the ultimate scour is 11.3 meters. In the wide pool, the bed is 3.6 meters below the crest at the low flow and 10.1 meters below at the high flow; so the ultimate scour in the wide pool is 6.5 meters.

At the high flow rate the supply of sediment from upstream is 900×10^6 metric tons per year, and this would finally be the sediment transport rate throughout the system when equilibrium is established. Initially at the tail of the first long pool, the transport rate is 5760×10^6 metric tons per year; therefore, scour would occur, deepening and lengthening upstream until a depth of flow of 19.8 meters is attained. At the head of the next narrow pool, the capacity initially is 6660×10^6 metric tons per year, and this section also scours. The tail end of this second pool has a capacity of 9000×10^6 metric tons per year; the entire pool scours and in the process steepens. The head of the next wide pool has a capacity of 3060×10^6 metric tons per year and it fills, while the tail end with a capacity of 5760×10^6 metric tons per year scours. The next narrow pool has a capacity of 6660×10^6 metric tons per year at the head and 9000×10^6 metric tons per year at the tail and fills at the head and scours at the tail.

After temporary equilibrium is approximately established, the capacity in the second pool would be less than it was initially, about 5500×10^6 metric tons per year, in the third pool about 4400×10^6 metric tons per year and in the fourth pool about 7800×10^6 metric tons per year.

The supply rate from the first pool would be less than initially and less than the capacity in the second pool; therefore, the second pool should continue

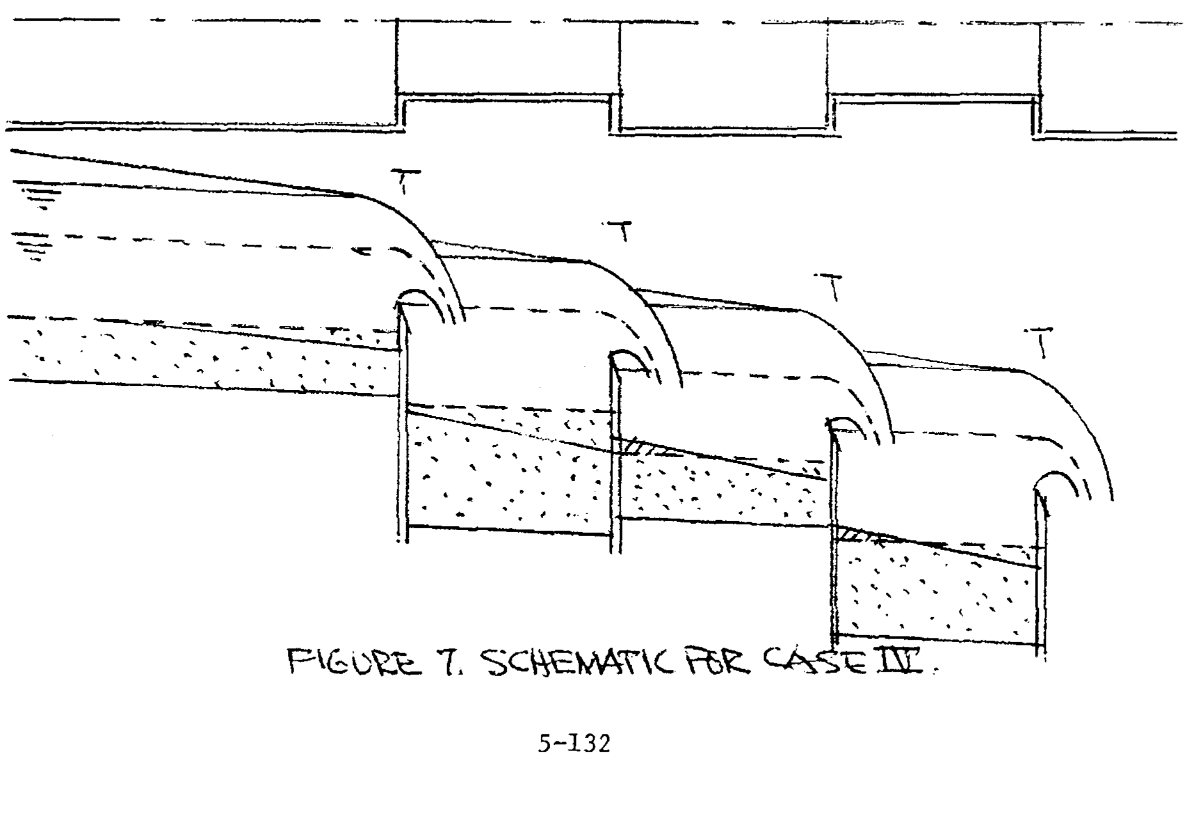

FIGURE 7. SCHEMATIC FOR CASE IV.

scouring. The capacity in the third pool would be less than the supply from the second pool and it would fill. The fourth pool would have a greater capacity than the supply and it would scour.

CASE V. A SIMPLE POOL-RAPID RIVER

The pool-rapid river shown in Figure 8 is a combination of the two previous cases; the wide (180 meter) pools have a triangular control, the narrow (120 meter) pools have a parabolic control. At a flow of 1400 m^3/sec, the head on the triangular control is 4.9 meters, the equilibrium depth in the wide pool is 8.5 meters, and the bed is 3.6 meters below the crest. At that low flow the head on the parabolic weir is 4.8 meters, the depth in the narrow pool is 11.0 meters, and the bed is 6.2 meters below the crest.

At a flow of 5600 m^3/sec the head on the triangular weir is 9.7 meters, the equilibrium depth in the wide pool is 19.8 meters, and the bed will finally scour 6.5 meters to be 10.1 meters below the crest. The equilibrium rate of sediment transport would be 900×10^6 metric tons per year, but initially just above the control the depth will be 13.3 meters and the capacity will be 5760×10^6 metric tons per year. At the head end of the pool the depth will be 15.0 meters initially and the capacity will be 3060×10^6 metric tons per year.

At the high flow the head on the parabolic weir will be 9.1 meters, the equilibrium depth in the narrow pool will be 27.1 meters, and the bed will scour 11.8 meters to be 18.0 meters below the crest. The initial depths at the head and tail of the pool will be 18.3 meters and 15.3 meters, and the capacities will be 6300×10^6 and 17100×10^6 metric tons per year.

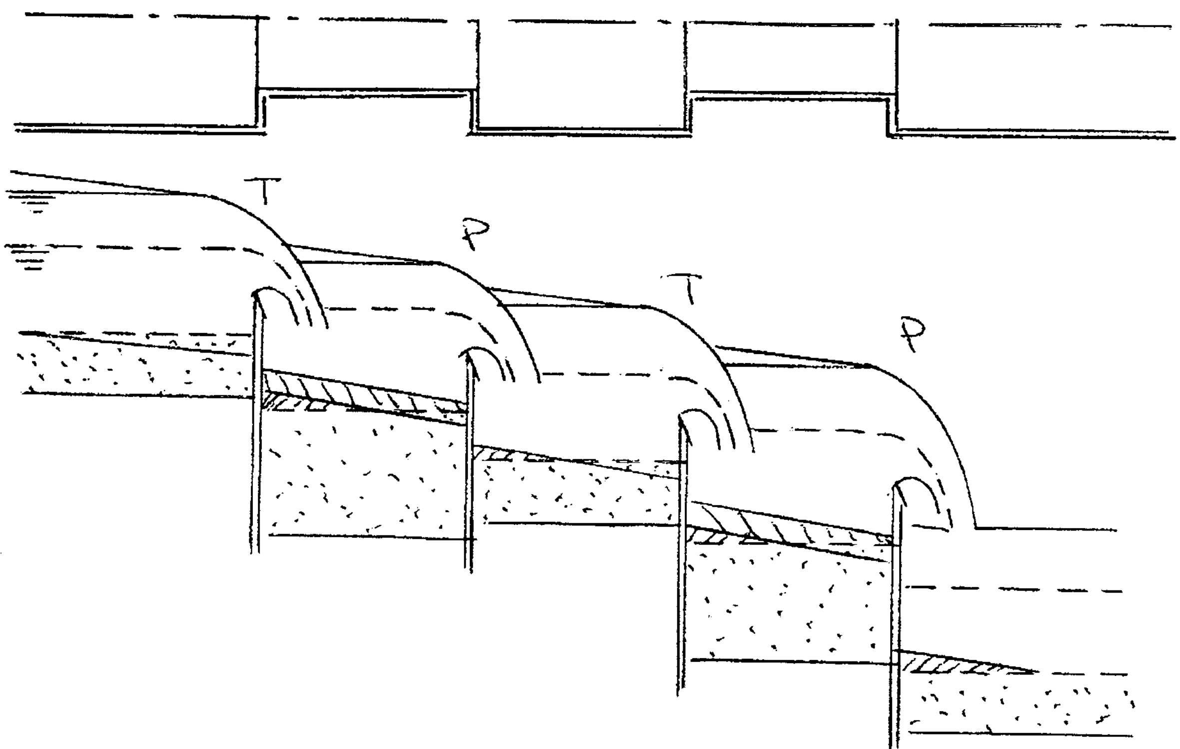

FIGURE 8. THE POOL-RAPID RIVER

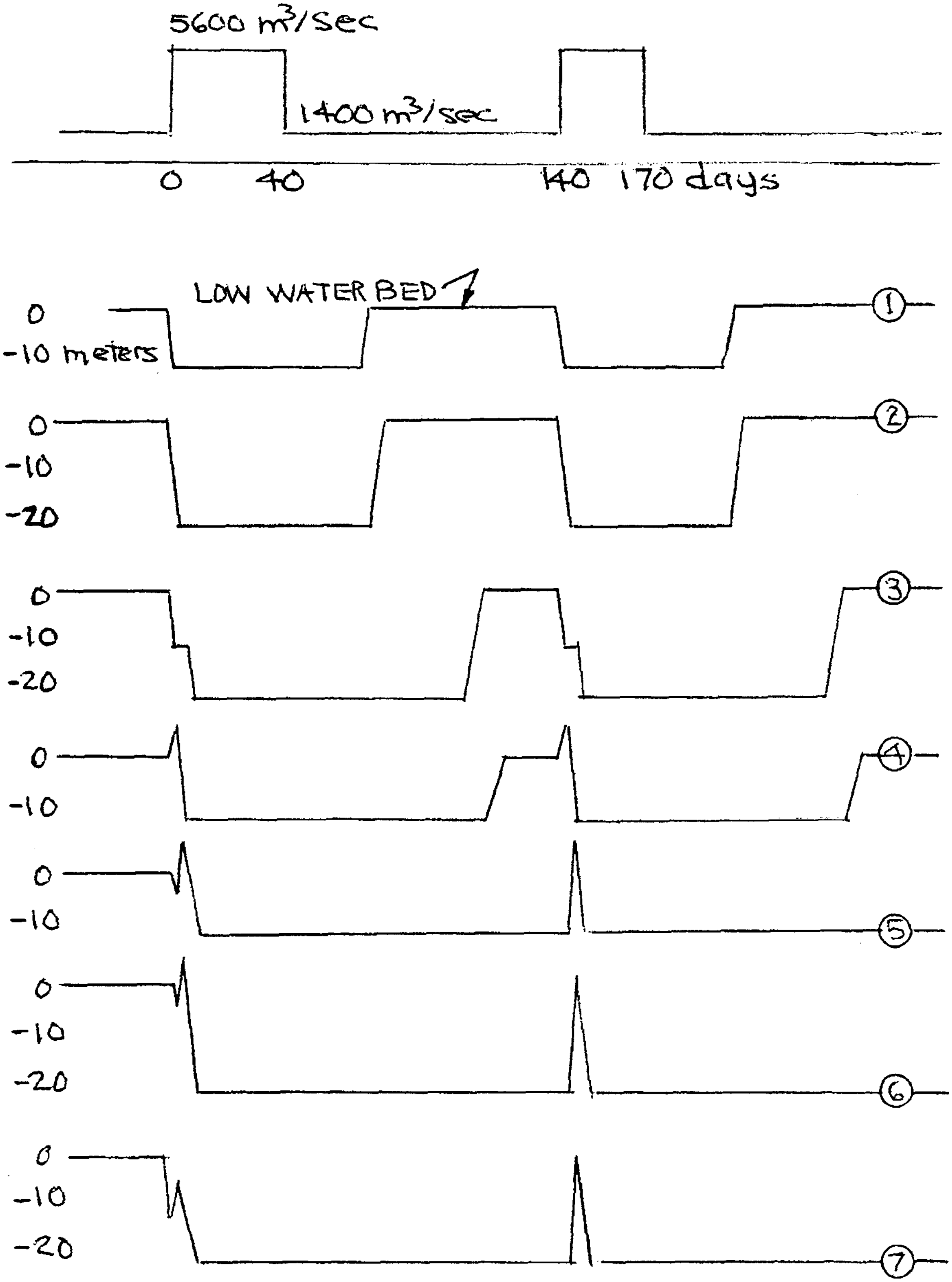

FIGURE 9. SCOUR & FILL vs. TIME

Figure 9 shows the bed elevations just above and just below the controls as a result of a hydrograph of two floods of 5600 m^3/sec of 40 days and 30 days duration, separated by a low flow period of 100 days at 1400 m^3/sec and that same low flow preceding and following the floods.

The first long pool scours out above the control and head cuts back. The scour hole fills from upstream and therefore there is a delay before the bed rises at the control. The second pool scours out both from the head and the tail. The lower scour hole enlarges faster but does not scour out to equilibrium depth as more than the equilibrium transport rate is being supplied to it. The upper scour hole goes down to the limit and when the two holes meet, the upper one proceeds on downstream. The second pool must wait until the first pool is refilled before it can fill.

The third pool first fills at the head, then scours; at the tail it first scours, then fills, then scours again. At the intermediate low flow only part of the pool fills; as a result on the second flood at the tail end a short-lived delayed hump appears and disappears. These humps, slightly delayed, appear and disappear at both the head and the tail of the fourth pool during the second flood. The head of the fourth pool scours, fills, and scours just like the tail of the third pool. Similar action occurs at the tail of the fourth pool but the fill does not come up to the low water bed level.

With a more complex river and a more complex hydrograph, it is easy to imagine very erratic patterns of scour and fill with time and space.

QUALIFICATIONS

Real pool-rapid rivers are more complex than considered herein. Pools can vary in width along their length; head-discharge relationships are not necessarily simple power functions and controls can be drowned out at high discharges. Tributaries can add flow at intermediate points; and can change the relative concentration and the composition of the sediment load.

Rock falls, cobbles and boulders can effectively riprap the bed at some elevation so scour cannot proceed and so an increased sediment load cannot be obtained. Analysis in such a case would have to consider the characteristics of clear-water scour.

Above and below the rapids the flow would be quite nonuniform, the supercritical flow leaving the rapids is likely to form a jet plunging to the bottom of the stream. Especially after the rapid, a local scour hole is to be expected, perhaps followed by a stationary dune. All such considerations have been ignored in this study.

CONCLUSIONS

The seemingly erratic behavior of pool-rapid rivers is subject to rational explanation. Because with increasing discharge, the equilibrium depth increases faster than does the head on the control, there will be scour in all pools if the flood lasts long enough. With decreasing discharge, of course, the opposite ultimate result—fill—is to be expected. Fill, however, takes much longer to occur than scour because the difference between sediment supply and capacity to transport sediment is much smaller.

During the active phase of scour (or fill) the seemingly erratic behavior becomes apparent. Sections that will ultimately scour may first fill because scour upstream supplies a temporary unusual high load of sediment. Temporary differences in capacity for transport can be due to differences in pool width or to differences in controls. A pool which follows several pools may fill and scour several times before scouring to equilibrium depth, because no pool can make its final move to equilibrium until all pools above it have reached equilibrium.

If the hydrograph is not long enough, the scour or fill to the equilibrium condition will be interrupted, and holes or humps will be left in the system. These holes and humps will tend to move through the system.

The normal river with contractions and expansions but without periodic controls (rapids) behaves similarly except that the reaches are linked hydraulically and the scour and fill takes place simultaneously throughout the system.

The combinations of sediment supply and discharge (and sediment composition) impose a requirement for a certain slope to the stream. Width, therefore bank erodibility, has only a slight influence on the slope. The roughness, or Manning n, has a considerable influence but is measurable (or estimable). Therefore, slope and flow should allow an estimate of watershed sediment yield of bed material.

ACKNOWLEDGEMENTS

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