MODELING STORM AND TILE DRAINS IN A MULTI-DIMENSIONAL HYDROLOGIC MODEL

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Abstract: GSSHA is a two-dimensional finite difference hydrologic model that has been used mostly in research of watershed flooding. Major processes that GSSHA is capable of modeling include overland flow (2D), stream flow (1D), infiltration (1D), groundwater (2D), and evapotranspiration. Recently a capability to model storm and tile drains was added to GSSHA. The storm and tile drain formulation is based on an implicit pipe scheme formulated by Dr. Zhong Ji [1998]. The formulation consists of a network of superlinks. The superlinks can be looped and interface with the overland flow, stream flow, and groundwater models to produce a spatially explicit, fully coupled storm and tile drain model. An example storm drain application is shown that demonstrates the effect of storm drain placement on the watershed response.

Forward and Acknowledgments: Much of the storm drainage model development and theory is taken directly from Jonathan Zahner's M.S. thesis [Zahner, 2004]. The application section is provided as an example.

INTRODUCTION

There is no argument that flood magnitude and frequency increase as urban development spreads throughout a watershed. It is obvious that understanding this trend is of great social and economic importance. But what causes this change in hydrology is the source of much debate and numerous studies. Changes in urban runoff volume and flood peaks have historically been blamed on increases in impervious area. This theory was recently challenged by a study in and around Charlotte, North Carolina [Smith et al., 2002]. The conclusion by Smith et al. [2002] was that the increase in storm drainage connectivity and hence hydraulic efficiency played the greatest role in increasing flood magnitudes. The inability to explicitly simulate storm drainage networks is seen as major limitation in the application of U.S. Army Corps of Engineer's distributed hydrology model Gridded Surface Subsurface Hydrologic Analysis (GSSHA) to urbanized areas. To address this issue the SUPERLINKS [Ji, 1998] storm drainage scheme was added to GSSHA [Zahner, 2004]. To verify that the complete model was operating properly, Ji's [1998] test simulation for a simple six-pipe network was reproduced.

SUPERLINK THEORY AND INTEGRATION

Superlinks are series of links connecting junctions, and must have a junction on either end. A *junction* is defined as a point where two or more superlinks meet, or the unconnected end of a superlink (such as intake/discharge point of network). A *link* is a segment of a superlink connecting two nodes, and a *node* is a computational point in a superlink. The use of both nodes and links may seem redundant, but in fact is quite integral to the "staggered grid" technique employed in SUPERLINK and is discussed later in detail.

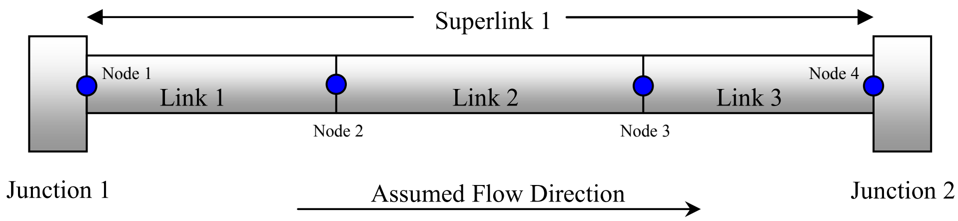


Figure 1. SUPERLINK junction, link, and node nomenclature.

Inflow is allowed at junctions and nodes via two primary structures. The first structure is a culvert, which captures a natural stream channel, and only is possible at a junction. The second is any type of grate/curb opening in a roadway, and is possible at either a junction or a node. Discharge can occur from a flooded manhole, drop inlet, or an outlet pipe (node or junction), and junctions may discharge directly into a channel. The superlink may also communicate directly with the groundwater model, allowing inflow along the length of each link, which is treated as an input at the next node.

Modeling Theory: The central equations solved in this model are the conservation of mass (Equation 1) and the de St. Venant equation of motion (Equation 2). This pair of nonlinear partial differential equations take the form of

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_0 \tag{1}$$

$$\frac{\partial Q}{\partial t} + \frac{\partial Qu}{\partial x} + gA\left(\frac{\partial h}{\partial x} - S_0 + S_f + S_L\right) = 0 \tag{2}$$

where A = flow cross-sectional area, Q = discharge, h = depth, u = velocity, S_0 = bed slope of conduit, S_f = friction head loss slope, S_L = local head loss slope, q_0 = lateral flow to conduit, g = gravitational constant, x = distance, and t = time.

The two fundamental equations (Equations 1 & 2) are applied on sections of a conduit segmented by computational nodes. Conservation of mass is represented by Equation 1, and is applied across a node. The staggered grid approach requires the conservation of momentum (Equation 2) to be applied on a different control volume. The layout of these volumes is shown in Figure 2.

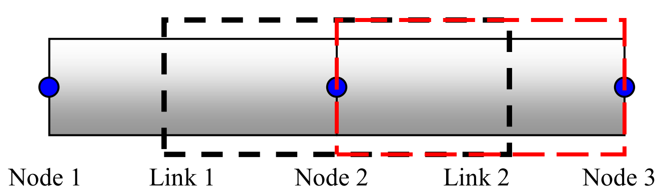


Figure 2. SUPERLINK staggered grid computational scheme.

The control volume shown in short dashes (black) illustrates the continuity equation for node 2, while the long dashed (red) envelope indicates the momentum equation for link 2.

The de St. Venant equations of motion only apply to free surface flow. During intense events, subsurface systems commonly flow full and under pressure. A common solution is to employ the "Priessmann slot" to extend the free surface equations to conduits flowing full. This slot area is not used for flow calculations, but merely to pressurize the conduit still being modeled by open channel flow equations.

Linearized Equations: To solve the partial differential equations, they must be discretized over their respective control volumes. Thus, unsteady terms such as flow rate and depth become time dependent variables. The discretized continuity equation with indices referring to Fig. 2 becomes

$$Q_2^{t+\Delta t} - Q_1^{t+\Delta t} + \left(\frac{B_2 \Delta x_2}{2} + \frac{B_1 \Delta x_1}{2} + A_{S1} \right) \frac{h_2^{t+\Delta t} - h_2^{t}}{\Delta t} = Q_{01} \tag{3}$$

The momentum equation takes a similar form

$$\begin{aligned}\left(Q_2^{t+\Delta t} - Q_2^{t}\right)\left(\frac{\Delta x_2}{\Delta t}\right) + x_3 Q_3^{t+\Delta t} - x_2 Q_2^{t+\Delta t} + gA_2\left(S_{f2} + S_{L2}\right)\Delta x = \\ gA_2 S_{02}\Delta x + gA_2\left(h_2^{t+\Delta t} - h_3^{t+\Delta t}\right)\end{aligned} \tag{4}$$

The only new term in these equations is B, or the top width of flow area. Subscripts refer to the link or node number, and superscripts denote either the current or future time step (current if not marked).

Boundary Conditions: As with any modeling problem, a set of boundary conditions must be applied to the extents of the network. With regards to the SUPERLINK model, these boundaries are located at the ends of each superlink, or junctions. The first component of the junction boundary is the water surface elevation (head). Junction heads may be known or unknown, as determined by the actual network configuration. A known junction head may be controlled by something external to the model, such as a reservoir at the network outlet. This feature would create backwater pressure propagating upstream, thus affecting flow upstream. Unknown junction heads occur at internal connections of two or more superlinks. Junctions representing an intake structure at the start of a superlink could also have an unknown head.

Flow into and out of these junctions, whether of known or unknown head, is governed by end condition boundary equations. Inlet entrance geometry governs pipe flow in steep channels, and exit properties can control in low gradient conditions. The end equations use the head in the junction as well as geometric variables to produce a set of coefficients for each inlet and outlet. The inlet and outlet coefficients by Ji [1998] were found to be unstable in certain situations and were reformulated as discussed in the entrance and exit hydraulics section.

Solution Technique: The implicit scheme is defined by a simultaneous solution to all unknowns in the system at each time step. Instead of computing the head at every internal point (junctions and nodes) as the model steps through time, only unknown junctions are part of the solution matrix. The reduction in the matrix size and thus computational demand is substantial. But the elegance of this routine is the way in which the unknown internal node depth and flow are incorporated into the junction matrix solution. Through a series of recurrence relations, the momentum and continuity equations are propagated throughout each superlink from one node to the next. This is done in both the forward and reverse directions in order to capture both positive and negative flow. The resulting coefficients become part of a relatively complex equation relating junction heads, superlink end conditions, internal node depth, internal pipe flow rate, and current timestep network inputs. Full details of the SUPERLINK scheme are presented in Ji [1998]. The solution technique to solve the resulting matrix is a generalized LU decomposition technique. The generalized LU decomposition technique is used because the matrix can, and often will, be a sparse random matrix.

Entrance and Exit Hydraulics: In order to calculate flow through the pipe network consideration must be given to the entrance and exit hydraulics of the pipes being modeled. The general equation for inlet-controlled flow is given as

$$Q = CA\sqrt{2g\Delta H} \tag{6}$$

where C is a geometric coefficient, A is the flow area, and ΔH is the difference in head between the supply reservoir (junction) and pipe (node 1, link 1). Ji (1998) had taken entrance boundary equations from other sources, and thus the derivation could not be easily followed. Re-deriving the superlink end equations from Equation 5 created an alternate set of boundary conditions. We define $\Delta H = H - h - Z_{inv}$ where H is the junction head, h is the depth at the first node, and Z_{inv} is the invert elevation of the first node. By squaring both sides of the flow equation we obtain

$$Q^2 = C^2 A^2 g\left(H - h - Z_{inv}\right) \tag{7}$$

The time varying Q is broken into the current time step and the future time step, and we solve for depth h, where $t+\Delta t$ it the future time step. (Equation 8) This process can be applied to the downstream end of a pipe as well to account for instances of backward flow. (Equation 9)

$$h_u = \frac{|Q_u| Q_u^{\,t+\Delta t}}{C_u^{\,2} A_u^{\,2} g} + H_u - Z_{inv,u} \tag{8}$$

$$h_d = -\frac{|Q_d| Q_d^{\,t+\Delta t}}{C_d^{\,2} A_d^{\,2} g} + H_d - Z_{inv,d} \tag{9}$$

The subscripts *u* and *d* in Equations 8 and 9 refer to the depth either upstream or downstream.

As with the pipe entrances, pipe exits were modified from Ji's [1998] algorithm to more accurately model various flow regimes. In exit hydraulics, four possible conditions are considered for pipes flowing less than full: a mild sloped channel, a steep sloped channel, a critical sloped channel, and a backwater case where head in the junction exceeds the head of critical depth in the pipe.

If the system is obeying conservation of momentum and the length of pipe is sufficient such that the friction slope is equal to the bed slope, the solved depth should be normal depth. Critical depth, however, must be calculated for the given flow rate and geometric variables. As the solution for critical depth is non-linear, a Newton-Raphson iterative solution is employed.

Changes to Model: The nature of equations 8 & 9 does not allow flow to move into the system when the area of flow is zero. It is therefore necessary to maintain a very small depth at the nodes even when flow is zero. A danger in imposing a depth is to create instability within the flow calculation, as physically these numbers should be simultaneously generated. Extensive testing found that an initial depth of 0.00001 m provided a stable minimum, allowing flow to commence without disrupting the mass and energy balance. This value is likewise imposed when inputs cease and a network drains, simply to keep the pipes "wet".

Linking to Surface Waters: Interaction between GSSHA and the drainage network is allowed to occur by controlling the end boundary conditions for SUPERLINKS. Inflow to the subsurface is permissible via culverts and grate openings in the roadway. The potential inflow to the drainage network (q_{in}) in each node is given as a percentage of the total ponded volume (V_{ponded}) in the GSSHA grid cell per time step (dt) and the number of grates per node (N = 1 to 4) as given by

$$q_{in} = \frac{N\alpha V_{ponded}}{dt} \tag{10}$$

where $\alpha = 1/N_{max}$. This conceptualization is necessitated by the fact that *GSSHA* planar grid cells are not typically small enough to accurately describe the micro-topography of curb depressions on crowned roadways where grates are typically located. It is further assumed that a cell with four grates would be capable of intercepting all ponded water for grid sizes on the order of 10 to 30 m. At each time step SUPERLINKS determines if there is sufficient capacity to accept from the inlet structures. If there is not sufficient space the flow will be forced to remain on the overland flow plane.

Any manholes containing heads greater than the ground surface elevation will result in a transfer of volume out of the storm drainage network to the GSSHA overland flow plane. Discharge to channels can occur from any specified outlet pipe and is explicitly calculated at each time step. For complete details of SUPERLINKS and subsequent GSSHA integration the reader is referred to Ji (1988) and Zahner (2004).

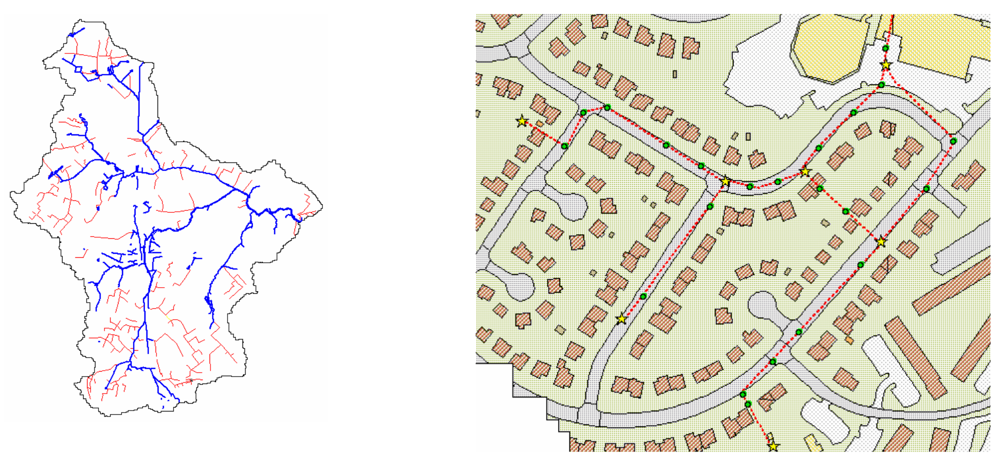
Linking to Groundwater: In order to use the superlink model as a tile drain model the addition of groundwater interaction has been implemented. A conductance value per meter length of pipe must be supplied to the model and based on that value and the head difference in the pipe and the groundwater in the cell flow is allowed to exit or enter the pipe.

EXAMPLE APPLICATION, DEAD RUN WATERSHED IN BALTIMORE, MARYLAND

It was necessary to model a watershed with a significant urban presence and subterranean drainage network to fully test the routines. A low gradient topography would provide situations of inundation and pressurized pipes. But perhaps most critical was the availability of a quality dataset including: rainfall records, stream flow records, digital elevation model (DEM), land-use and soil type coverages, stream channel, and storm drainage network data. Dead Run, a 14.3 km^2 watershed in Baltimore, Maryland, readily met these requirements. Impervious surfaces cover approximately 35% of Dead Run. Based on the land use and soil type GIS data sets, the GSSHA model was created at a resolution of 30 meters.

DEMONSTRATION SIMULATIONS

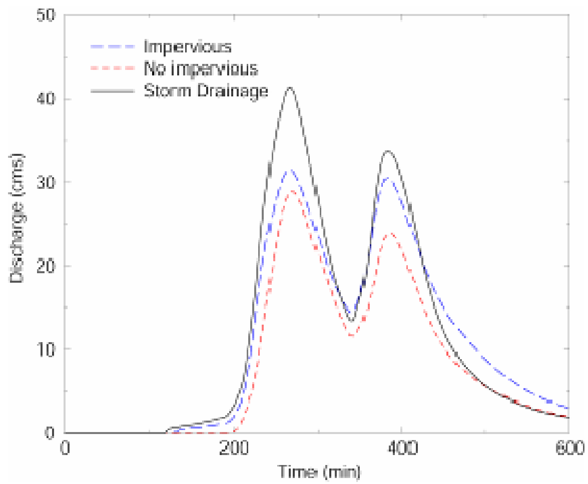
Since a storm drainage network will have its most pronounced effect during a moderate to high intensity storm, the precipitation event associated with Hurricane Isabel of September 18-19, 2003 was selected as the model test case. The storm dropped heavy rainfall on much of the east coast, including Maryland and the Baltimore area watershed of Dead Run. As is common in hurricane precipitation patterns, the area received two strong pulses of rainfall 150 minutes apart. The peak discharge recorded by the USGS gaging station at the outlet of the watershed was just under 40 m^3/s. Basin-averaged rainfall peaked at 53 mm/hr, but localized cells of intense precipitation were estimated by radar above 200 mm/hr. Thus the distributed nature of the rainfall input is as critical as the distributed land use and soil classification. This event was also selected because it allows calibration of GSSHA using the observed precipitation and stream records.

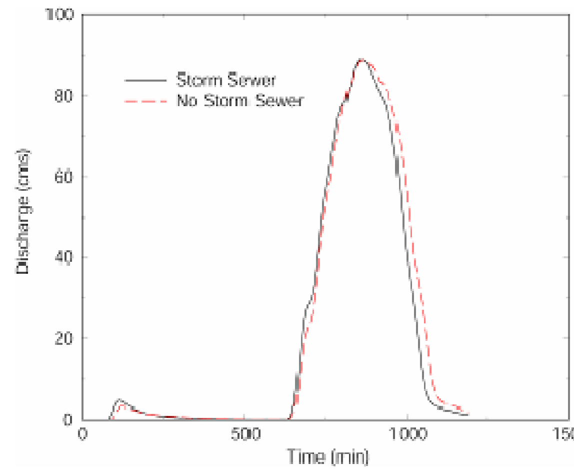


(a) (b)

Figure 3. The Dead Run watershed. The existing natural channel network in bold and storm drainage in thin lines in (a). The digitized drainage map with locations of inlet grates represented by dots is shown in (b).

Once the model was calibrated, the relative effects of impervious land cover versus the influence of the storm drains could be assessed. The model was run in three scenarios that sequentially added impervious areas and storm sewers. Case 1: no impervious areas, no storm sewer; Case 2: distributed impervious areas, no storm sewer; Case 3: distributed impervious areas with storm sewer network. These scenarios are shown in Figure 4a. All cases include the same channel network, detention basins, and culverts. The model was also run to determine the hydraulic effectiveness of the drainage network on an extreme event storm. The results are shown in Figure 4b.





(a) (b)

Figure 4. (a) compares the effects of imperviousness versus storm drains on the response of the watershed to a hurricane event. (b) shows the effects of the storm drains (versus no storm drains) on an extreme event.

<u>Results:</u> The demonstration simulations show that storm drain networks are significant hydrologic modifiers of a watershed. The effect of the storm drain network can even be greater that the addition of impervious areas to a watershed. Upon comparing the hydrographs in Figure 4a, a few conclusions become apparent. The addition of impervious area increases the total runoff volume but did not significantly adjust the time of the peak. The differences in the change to each of the two peak values from the addition of just the impervious area is probably due to limited infiltration capacity of the soil from low moisture content when the first peak of storm intensity passed over the watershed. The addition of the storm drainage network also shows a difference in the addition to the peak flow values. These differences are likely related to how the increased hydraulic efficiency of the drainage networks changes the storage of the water in the watershed.

The results of the second event (Figure 4b) indicate that there is an upper limit to the increased hydraulic effectiveness of storm drain networks. Under an extreme storm event the drainage network can quickly become overwhelmed and function at a limited capacity when compared to the total volume of runoff.

CONCLUSION

The addition of the superlink pipe network scheme in GSSHA allows for both storm and tile drain modeling. The implementation allows for fully coupled surface water and groundwater interaction in order to more accurately model the effects of drainage networks on the hydrologic response of watersheds. An example case was run on the Dead Run watershed in Baltimore, Maryland. The effects of both impervious area and the storm drain network were studied to determine the relative effect of each. For the event modeled (a hurricane on September 18-19, 2003), the storm drainage network had a greater impact on increasing the peak flow when compared to the impervious area. It was also demonstrated that there appears to be an upper limit to the increased hydraulic effectiveness wherein the drainage network can become overwhelmed when compared to the total volume of runoff.

ADDITIONAL INFORMATION

The study was conducted as an activity of the Regional Watershed Modeling and Management work unit of the System-Wide Water Resources Program (SWWRP). For information on SWWRP, please consult https://swwrp.swwrp.army.mil/ or contact the Program Manager, Dr. Steven L. Ashby at Steven.L.Ashby@erdc.usace.army.mil.

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